

Welcome to Geometry!

Geometry is one of the oldest areas of mathematics that deals with properties of space. We'll be learning about shape, distance, size, and position of objects.

We will discover many important relationships using the same tools that ancient mathematicians used many years ago: the *straight edge* and the *compass*.

The Point

A point is a location represented by a dot. It does not have any length, thickness, shape or size, it only has a position.



The Line

A line is straight, goes on forever, and has no thickness or depth.

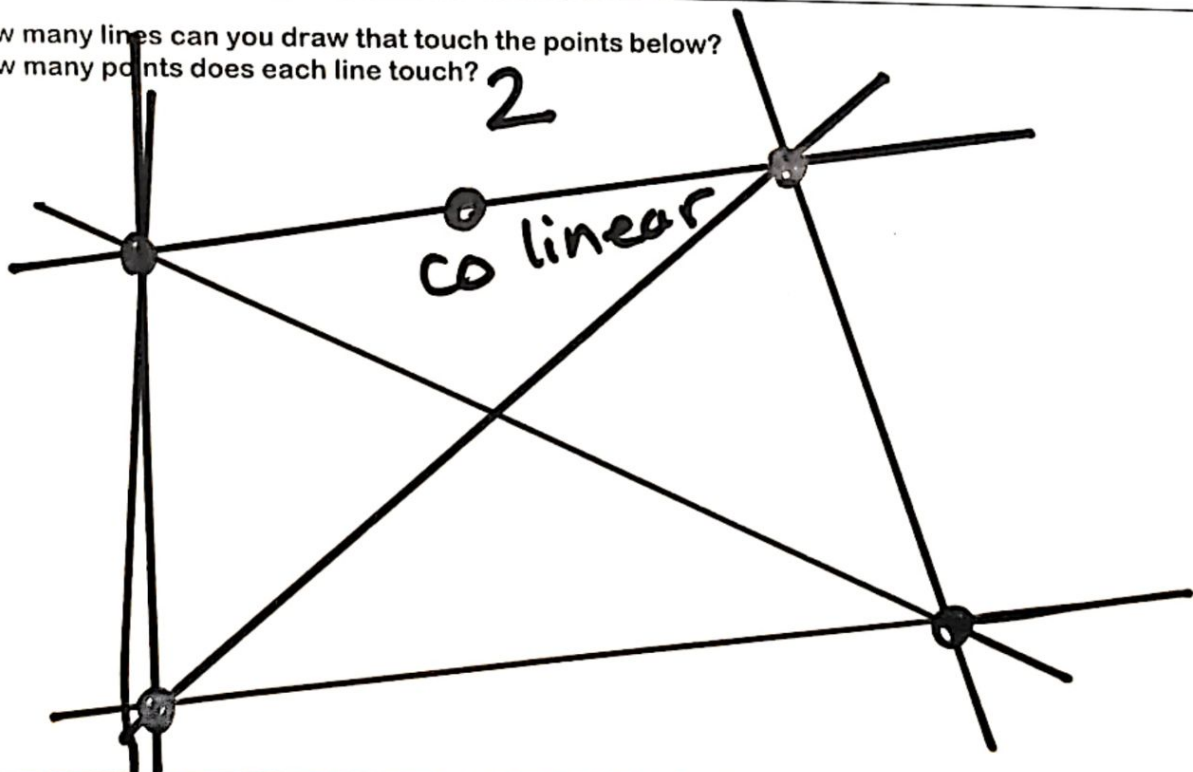


The **Straight Edge** is also called a *ruler* or a *straight tool*.

We use these to draw straight lines or to check a line for straightness.

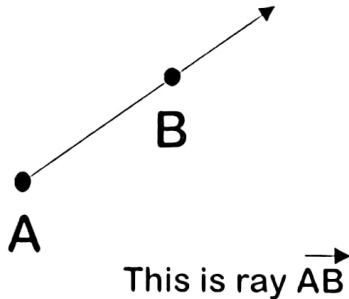


How many lines can you draw that touch the points below?
How many points does each line touch?



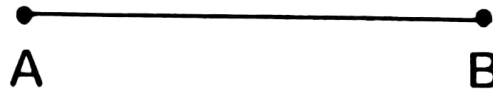
The Ray

The ray is like a laser – it starts at one end and goes on infinitely.



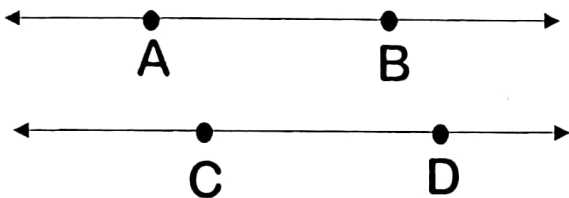
The Line Segment

The line segment starts at one end and ends at the other.



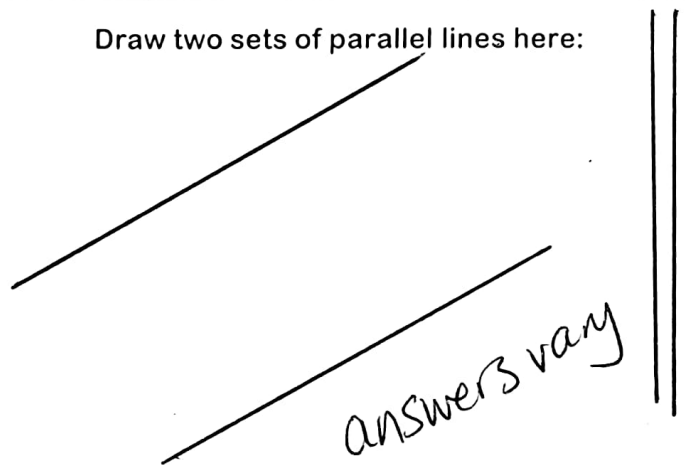
Draw a line that connects points A and B to make a line segment \overline{AB}

Parallel lines never meet.



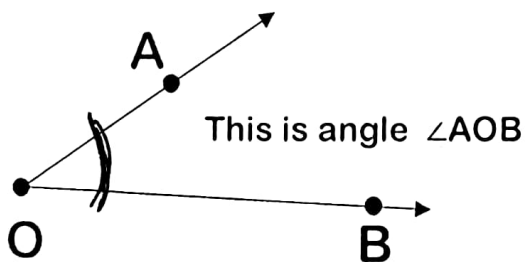
Line AB is parallel to Line CD: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

Draw two sets of parallel lines here:

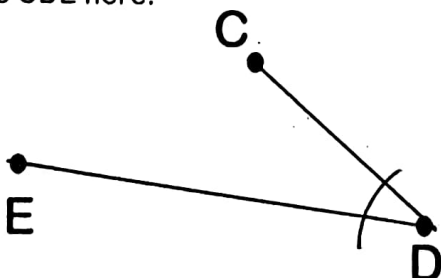


The Angle

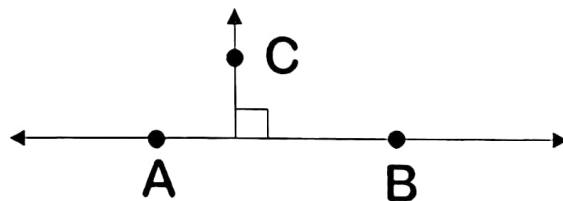
An angle is when two rays meet at a point, measured in "degrees".



Draw angle CDE here:

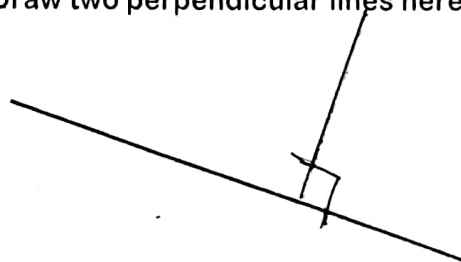


Perpendicular lines meet at a 90 degree angle.



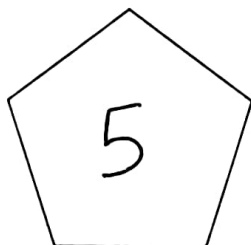
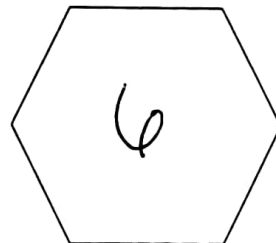
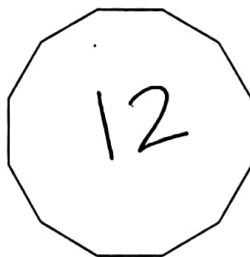
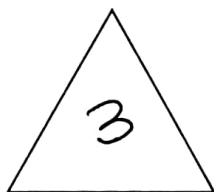
Line AB is perpendicular to Line CD: $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$

Draw two perpendicular lines here:



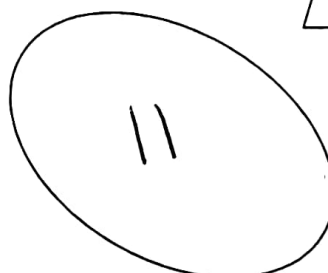
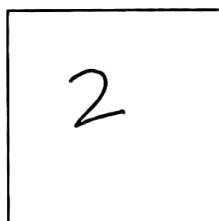
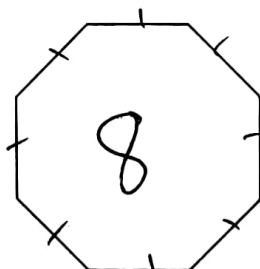
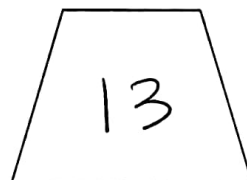
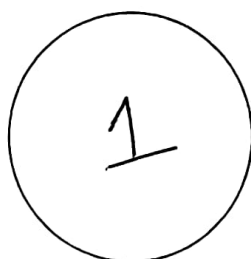
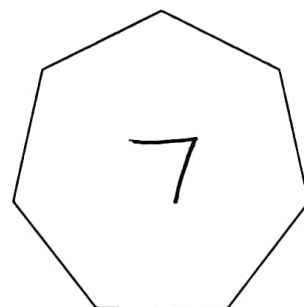
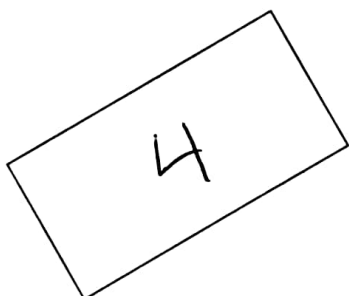
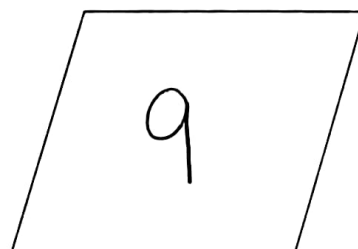
Basic Geometry Shapes

Three sides, four sides, five, six, seven, eight... how many do you know?



Write the number inside the corresponding shape:

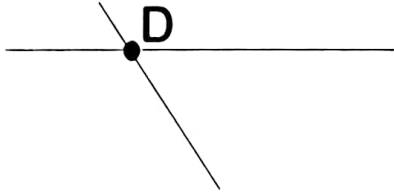
1. Circle
2. Square
3. Triangle
4. Rectangle
5. Pentagon (5 sides)
6. Hexagon (6 sides)
7. Heptagon (7 sides)
8. Octagon (8 sides)
9. Parallelogram
10. Decagon (10 sides)
11. Ellipse
12. Dodecagon (12 sides)
13. Trapezoid



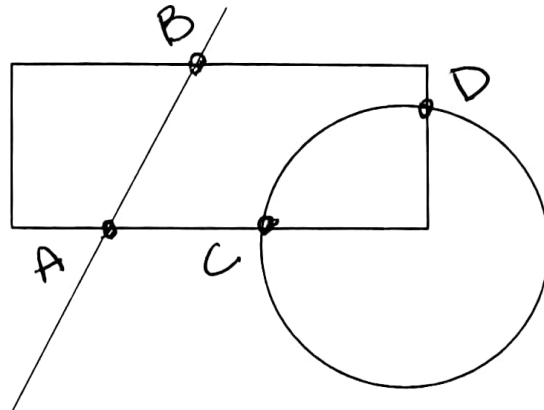
A polygon is a figure with at least three straight sides.

Point of Intersection

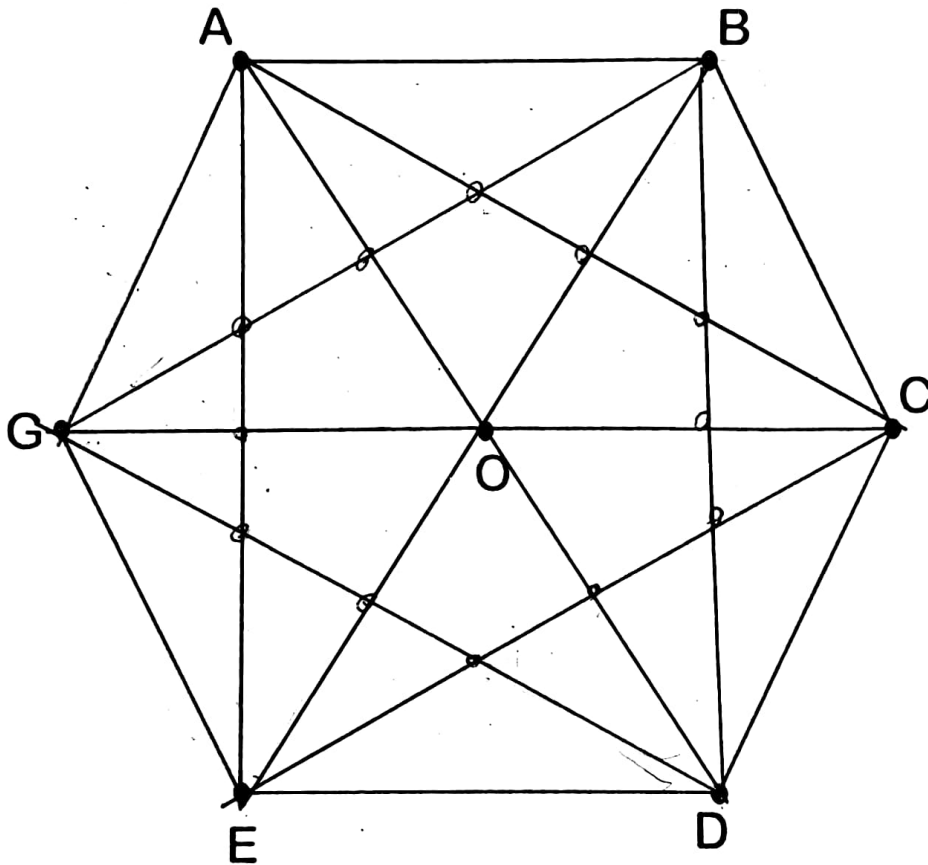
This is a point where two line segments meet or two lines meet.



Label all the points of intersection with different letters:



Draw all the line segments you can that connect A, B, C, D, E, F, G and O.



How many line segments did you draw? 30

How many intersections? 19

How many triangles? 18

How many quadrilaterals? 6

Are there any other shapes? hexagon, pentagon

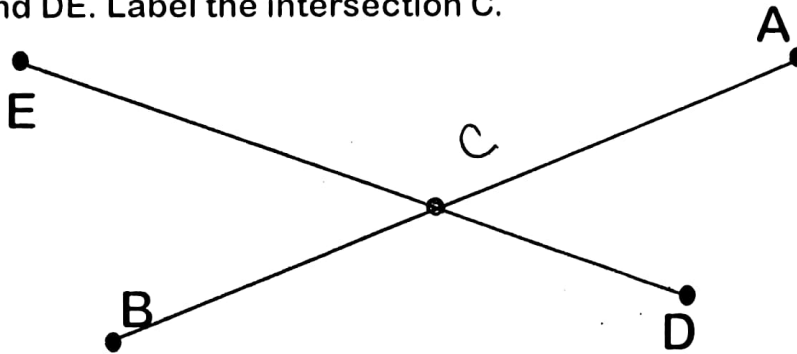
18 + ? you count!

A figure with six sides is a *hexagon*.

Intersections

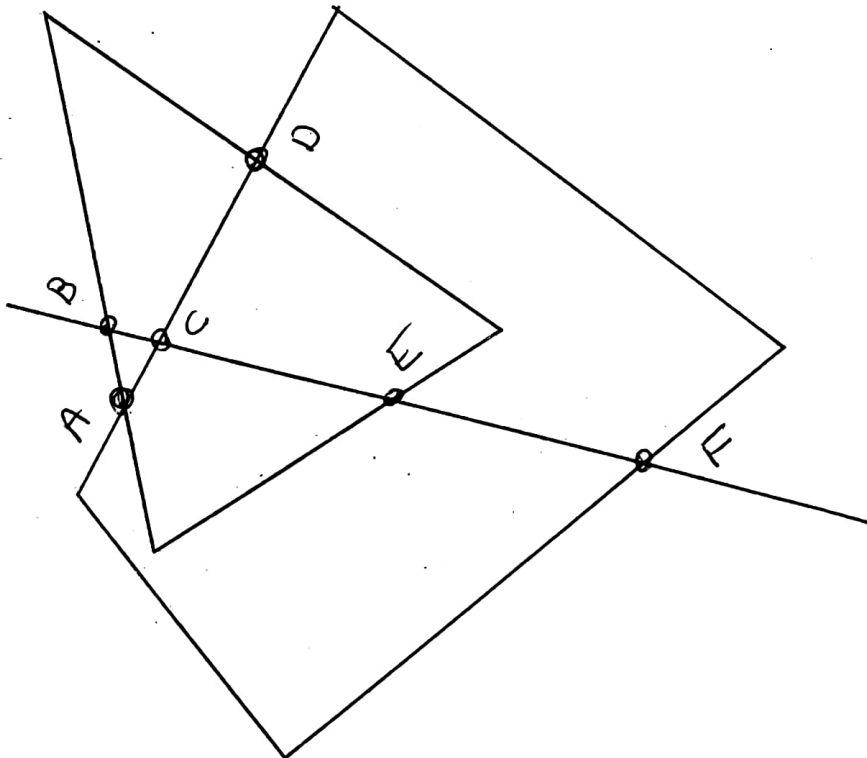
This is a point where two lines or curves meet.

Draw \overline{AB} and \overline{DE} . Label the intersection C.



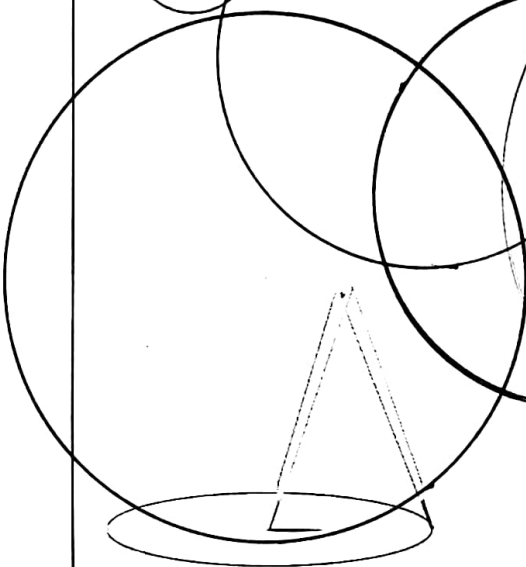
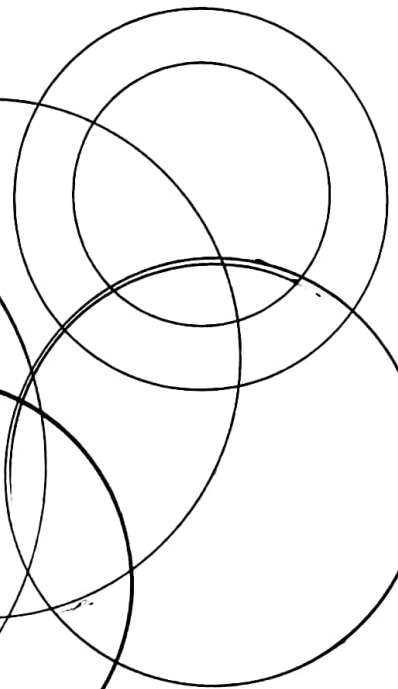
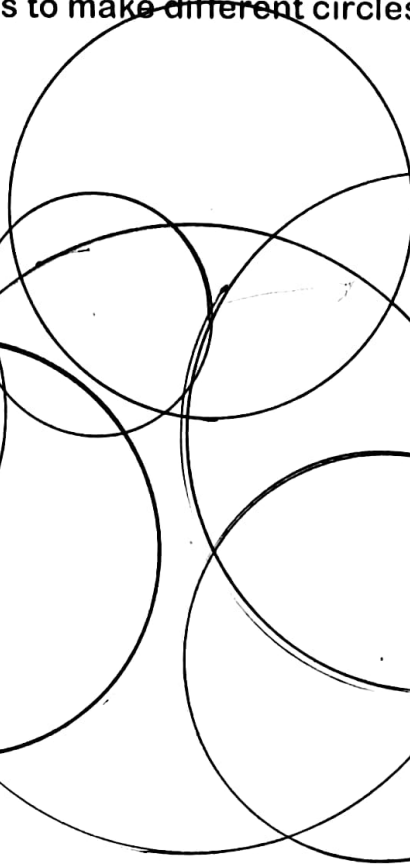
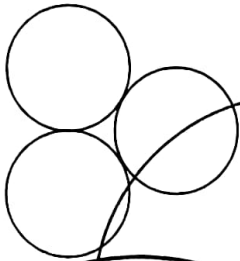
Draw a triangle, a quadrilateral, and a line so that they create six intersections. Label each intersection with different letters.

answers vary

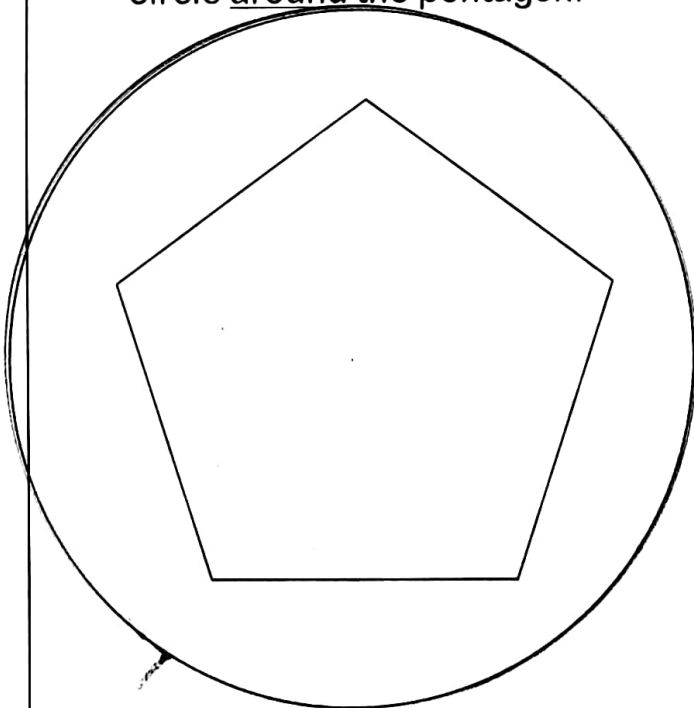


The Circle

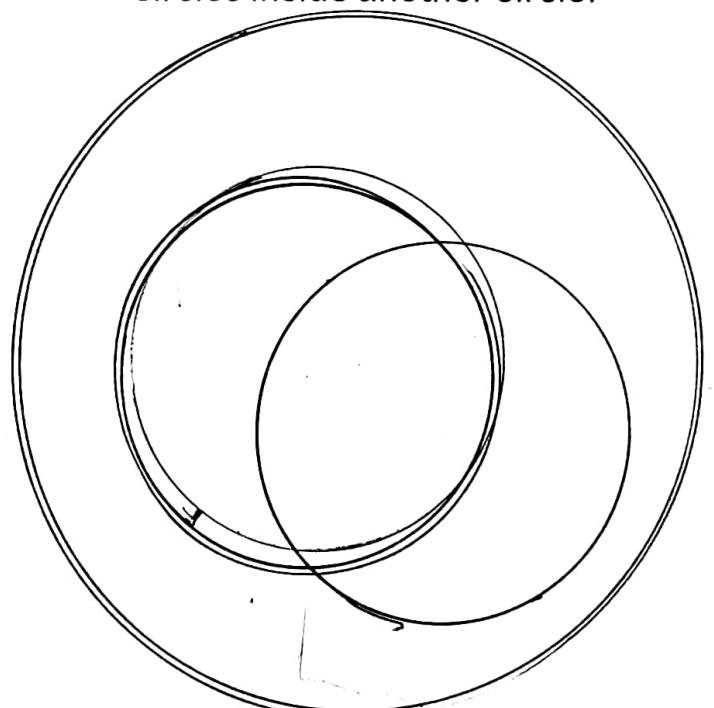
The circle is a round figure with no corners, no edges, and no flat lines.
Use your compass to make different circles and shapes!



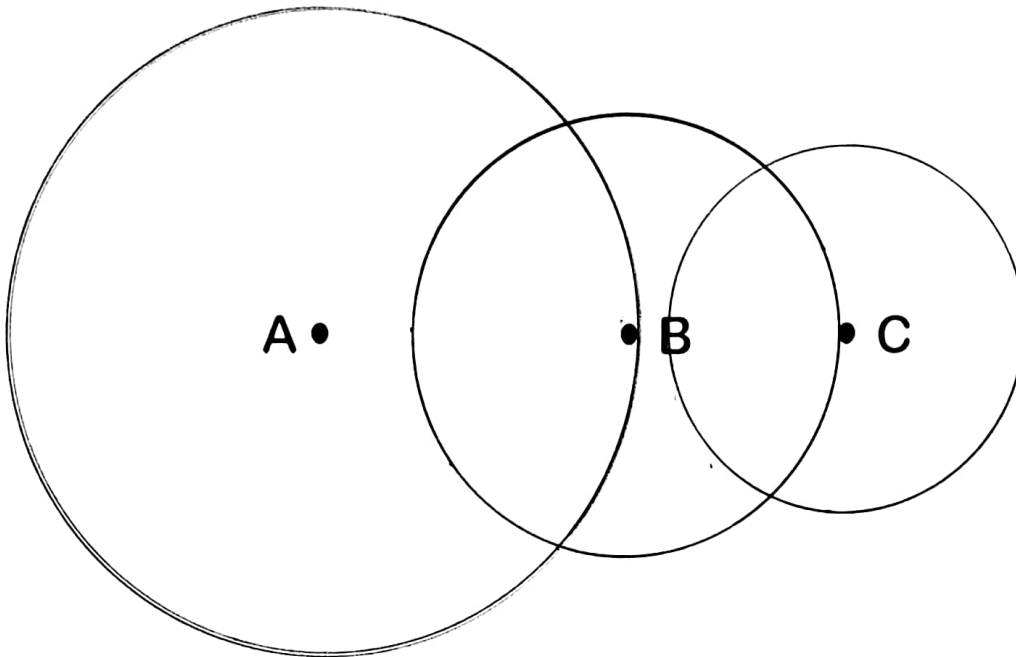
Use your compass to draw a circle around the pentagon:



Use your compass to draw a two circles inside another circle:



Draw a circle with the center at A, includes point B *on* the circle, with point C *outside* the circle.



Now draw a circle with the center on C which has the point B outside of it.

Draw a circle with center B that does ~~not~~ intersect ^{both of} ~~either of~~ the first two circles.

The Radius

This is ~~measured~~ *from* the center of a circle to any point *on* the circle.

Draw a circle with the center on Z.

Draw a line segment from Z to the circle. This line segment is your *radius*.

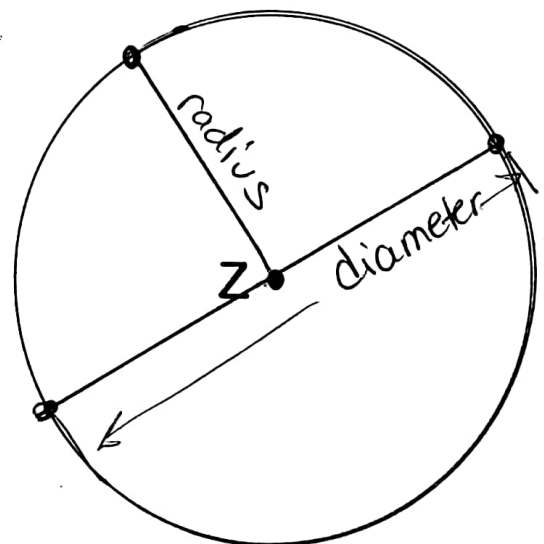
Measure the radius with your ruler:

3.5 cm

Now extend your line segment so it also touches the circle on the other side. This is your *diameter*.

Measure the diameter with your ruler:

7 cm



The Diameter

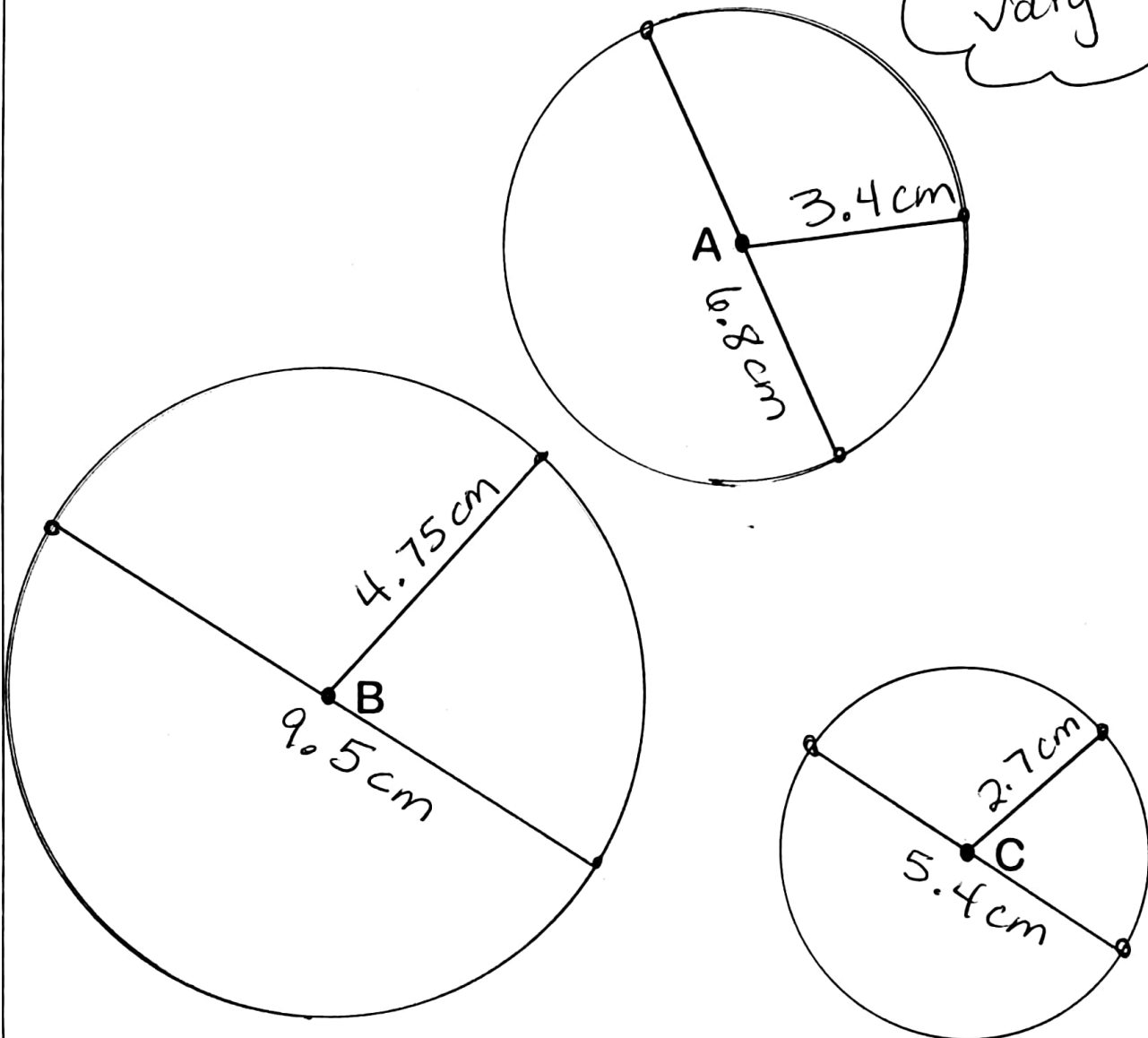
Any straight line passing through the *center* of the circle with endpoints *on* the circle.

Measuring Radius and Diameter

Use your ruler and measure the radius and diameter of each circle!

Draw three circles of different sizes at the points below.

Answers vary!



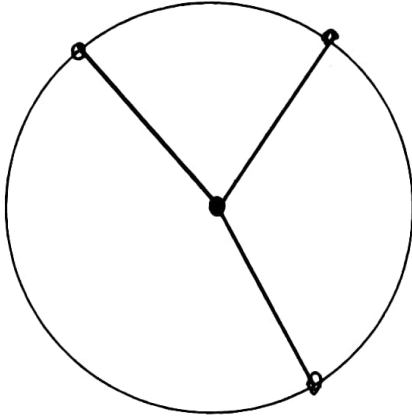
Draw a line through the center of each circle with your ruler, intersecting the circle twice.

Measure both the *diameter* and the *radius* of each circle (using inches or cm) and write these next to each circle.

Congruent

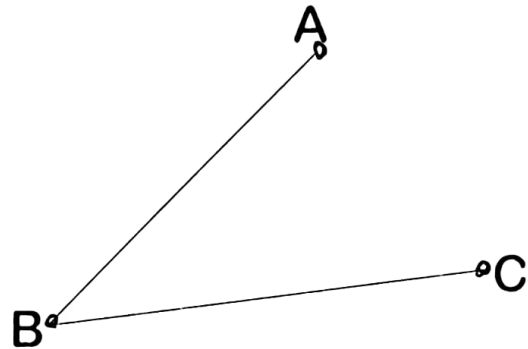
Congruent means to match *exactly* in both size and shape.

Draw three radii in the circle below.



Are these radii congruent? Yes

Use your compass to answer:



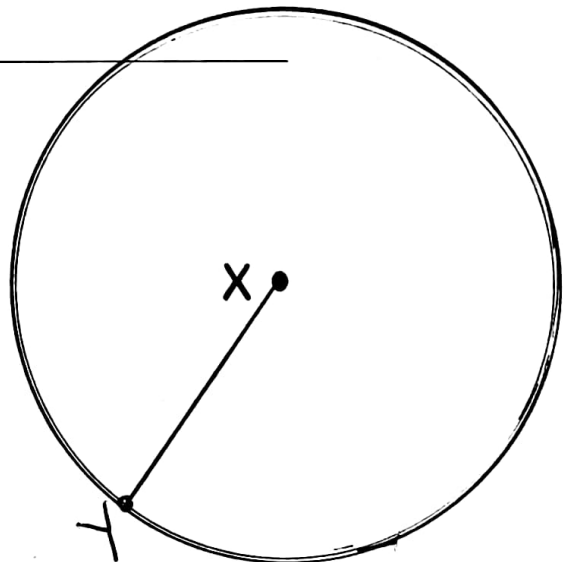
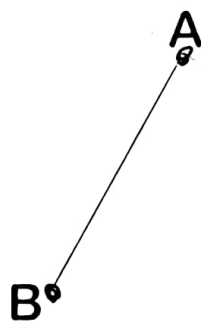
Are \overline{AB} and \overline{BC} congruent? No

Draw a circle with center A and radius \overline{AB} .

Now draw a circle with center X with a radius congruent to \overline{AB} .

Choose a point on the circle X and label this point as "Y". Draw \overline{XY} .

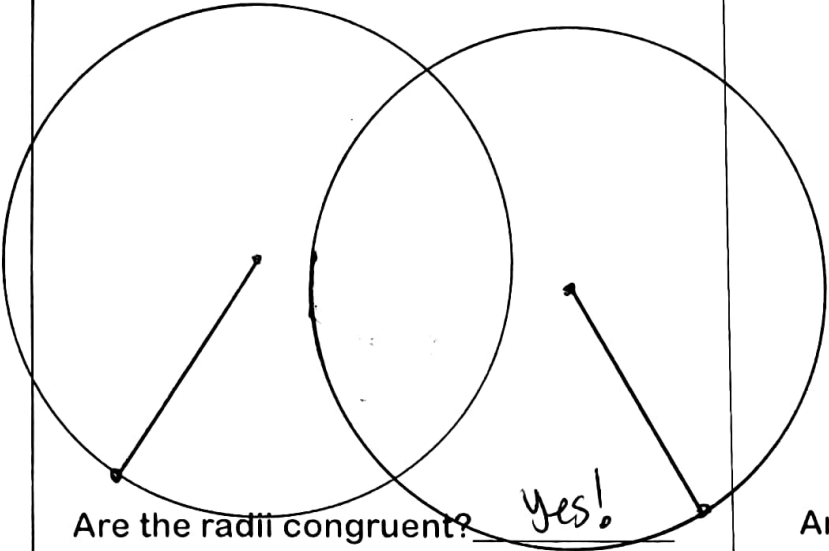
Are the two ~~circles~~ lines congruent? Yes



Congruent

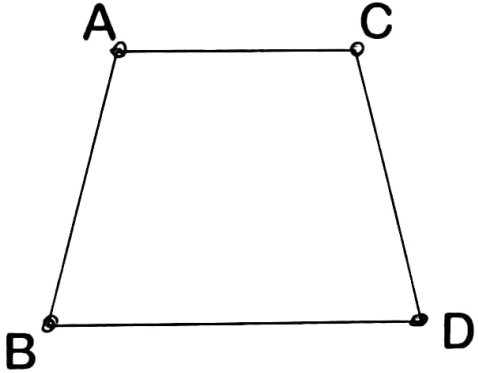
Congruent means to match *exactly* in both size and shape.

Draw two congruent circles.



Are the radii congruent? yes!

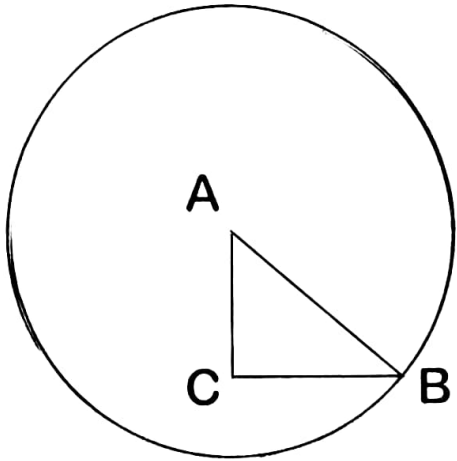
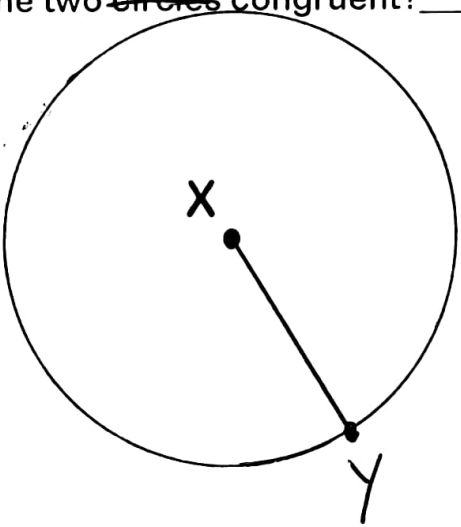
Use your compass to answer:



Are \overline{AB} and \overline{CD} congruent? yes

- ✓ Draw a circle with center A and radius \overline{AB} .
- ✓ Now draw a circle with center X with a radius congruent to \overline{AB} .
- ✓ Choose a point on the circle X and label this point as "Y". Draw \overline{XY} .

Are the two ^{lines} ~~circles~~ congruent? yes!



Measuring Angles with a Protractor

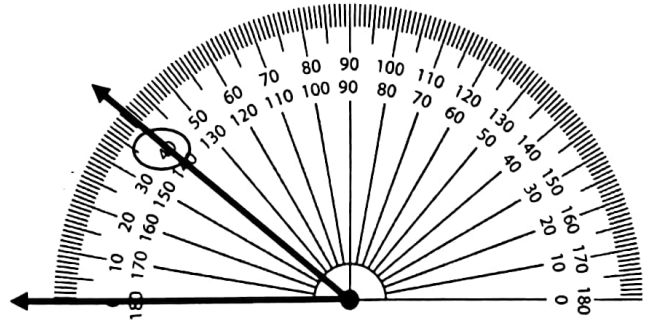
Angles are measured with a protractor. Angles are measured in *degrees*.

We measure line segments with a ruler.

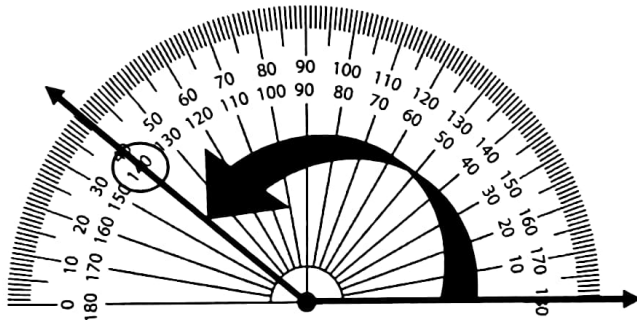
When we have an angle, we measure how “open” the angle is.

This line is “40 degrees”, which we write as 40° .

Notice that we are reading the top scale.



Always start at the zero and use the scale that matches with where you started.

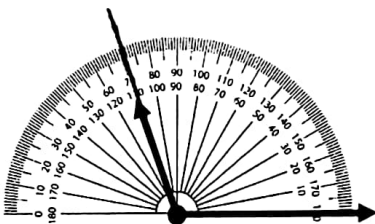


This time, we read the bottom scale because the zero is on the right.

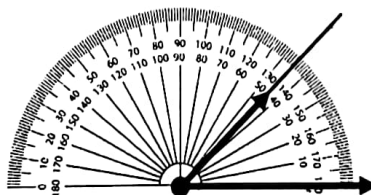
We read numbers increasing from zero on the lower scale.

This angle is now “140 degrees”, which we write as 140° .

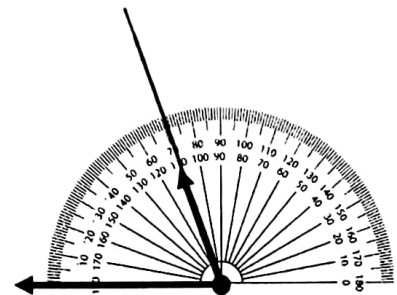
Measure the angle by extending the line through the scale:



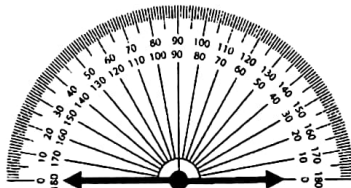
Angle: 110°



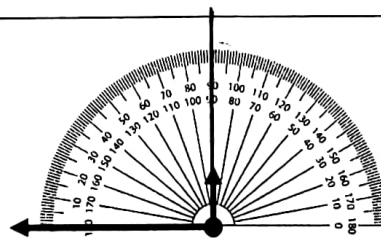
Angle: 45°



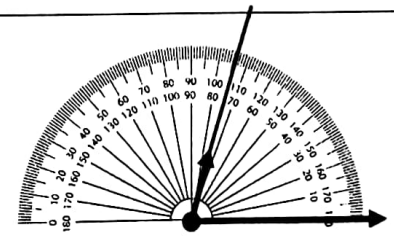
Angle: 70°



Angle: 180°



Angle: 90°



Angle: 75°

Measuring Angles with a Protractor

Measure each angle using your own protractor.

Angle: 134°

Angle: 72°

Angle: 115°

Angle: 83°

Angle: 35°

Angle: 74°

You can extend the lines first if they are too short to read on your protractor.

Angle: 108°

Angle: 45°

Angle: 70°

Angle: 180°

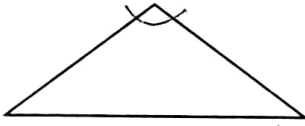
Angle: 90°

Angle: 74°

Special Properties of Triangles

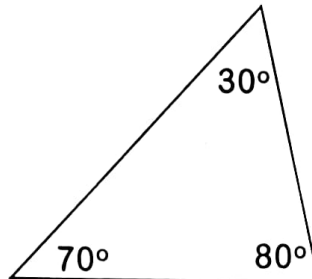
Triangles have properties we can use to find out everything about them!

A triangle has three sides and three angles.



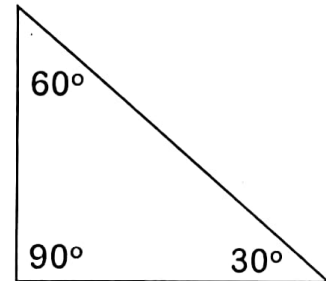
Obtuse triangles have one angle greater than 90° .

The sum of the angles always add up to 180 degrees.

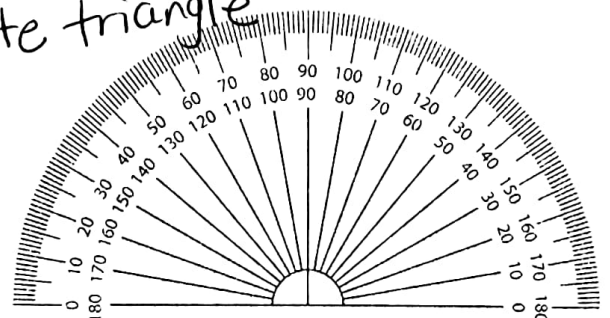
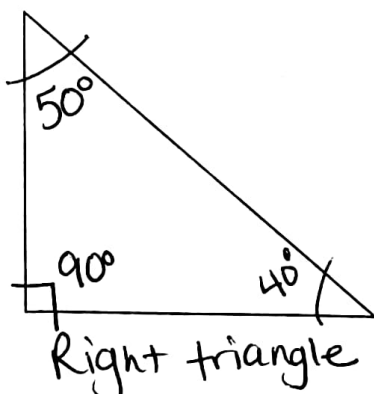
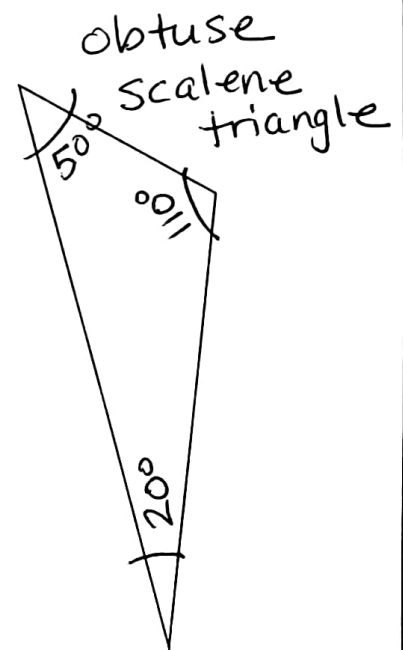
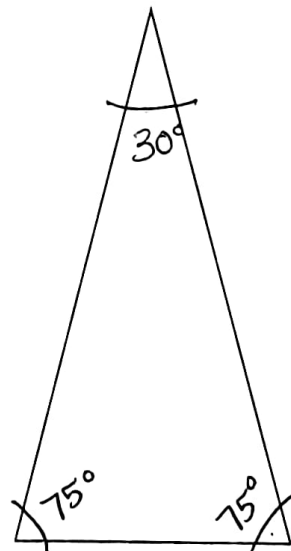
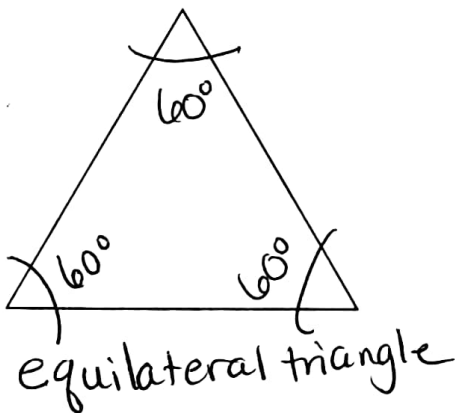


Acute triangles have all angles less than 90° .

A right triangle has one angle equal to 90° .



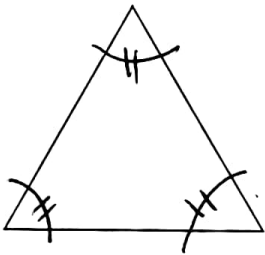
Use your protractor to measure the angles of each triangle. Label the type of triangle as acute, right or obtuse.



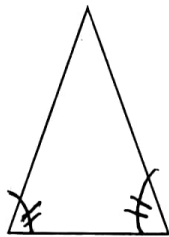
Special Triangles

These triangles show up often enough that they have their own special properties!

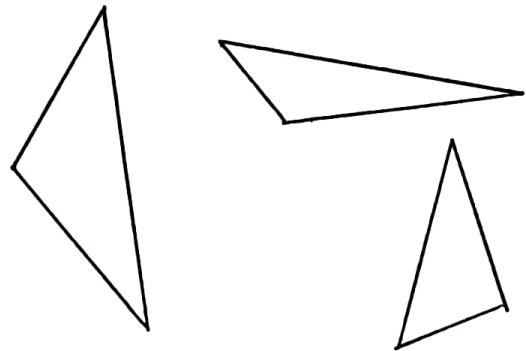
An equilateral triangle has three equal sides and three equal angles.



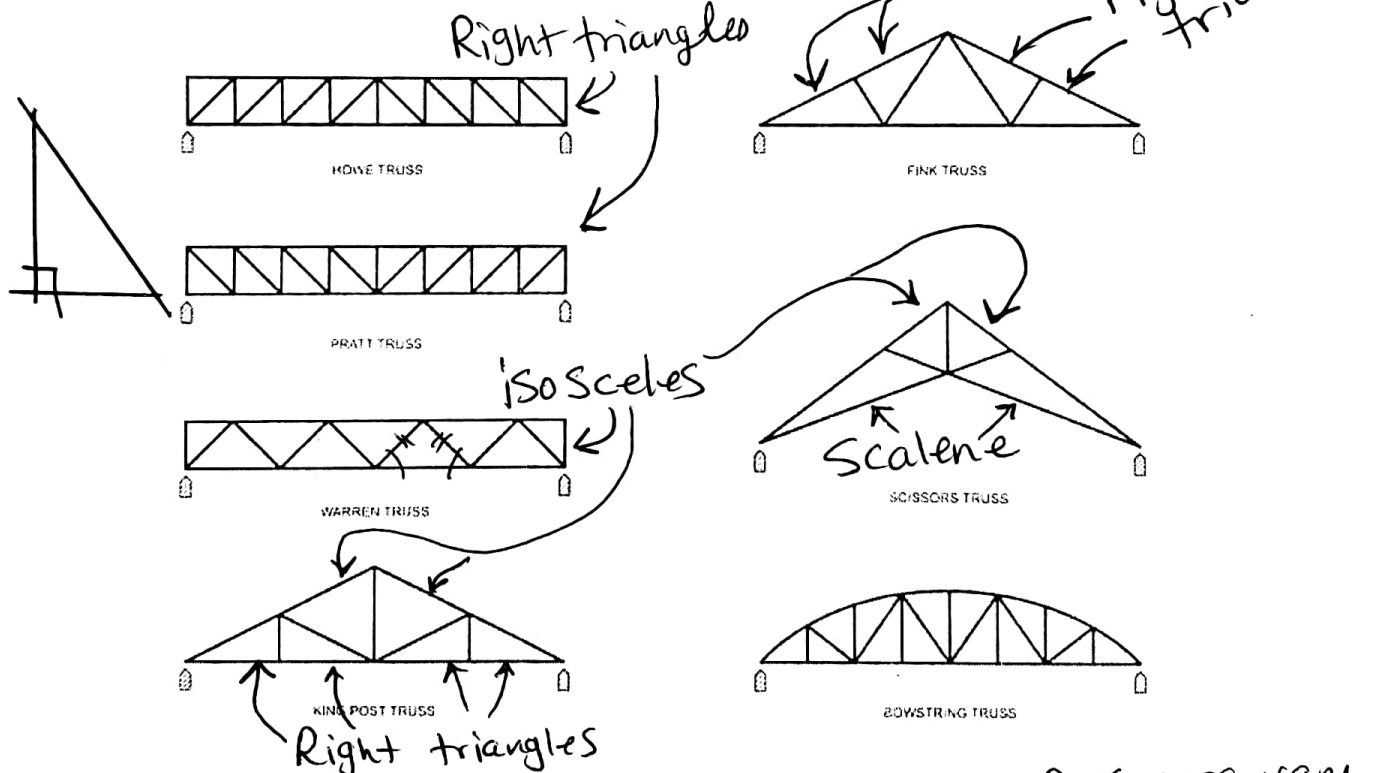
An isosceles triangle has two equal sides and two equal angles.



A scalene triangle has no equal sides and no equal angles. Draw three examples here:



These are steel truss roof frame designs. Which types of triangles do you notice in each?



Pick your favorite design. How many triangles do you notice? Answers vary

How many of the following are in your favorite truss?

Equilateral:

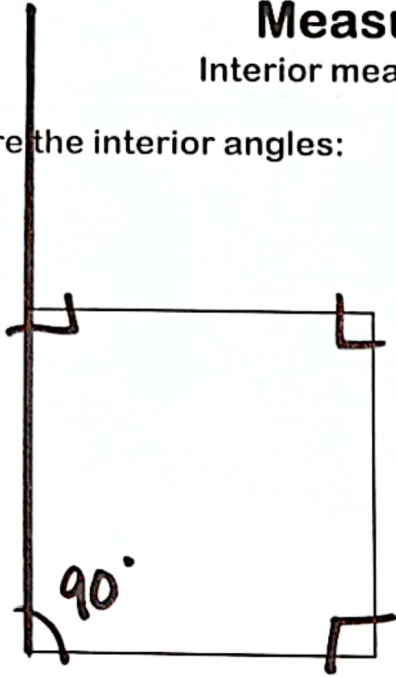
Isosceles:

Scalene:

Measuring Interior Angles

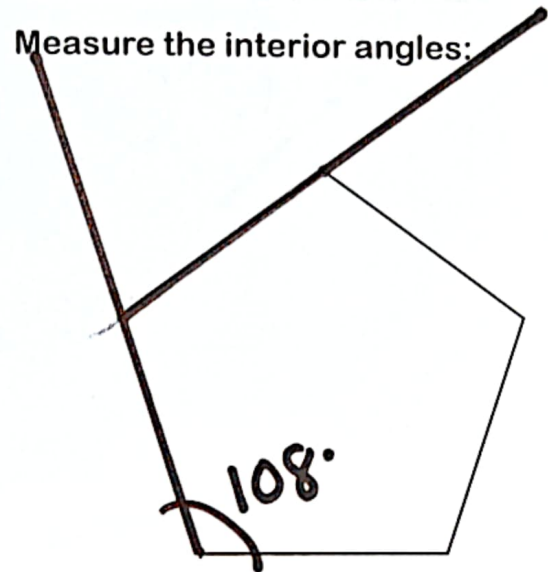
Interior means inside, exterior means outside.

Measure the interior angles:



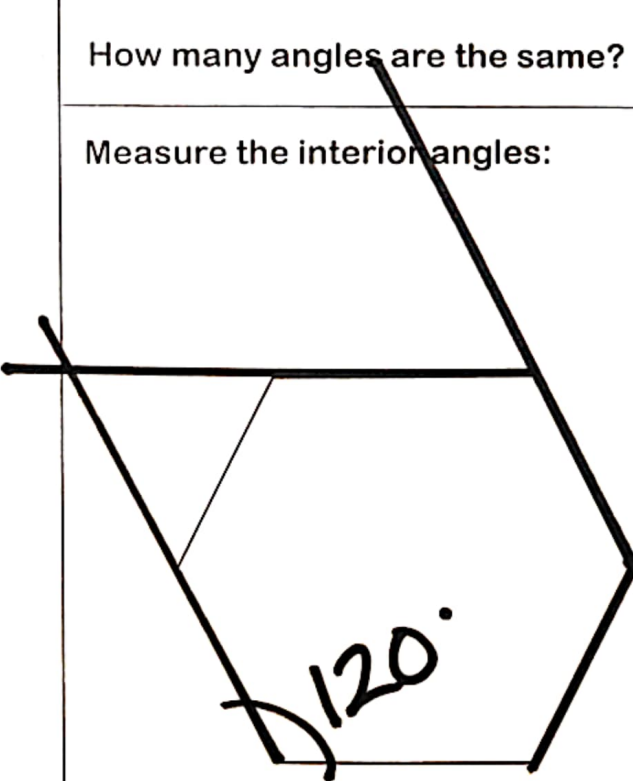
How many angles are the same? **4**

Measure the interior angles:



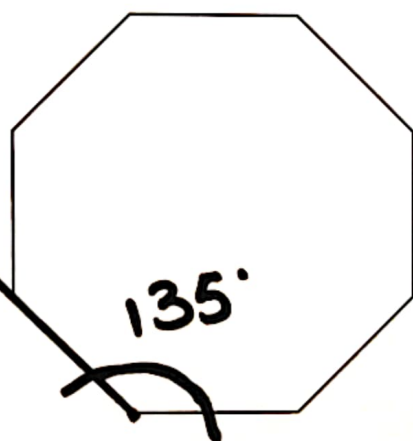
How many angles are the same? **5**

Measure the interior angles:



How many angles are the same? **6**

Measure the interior angles:

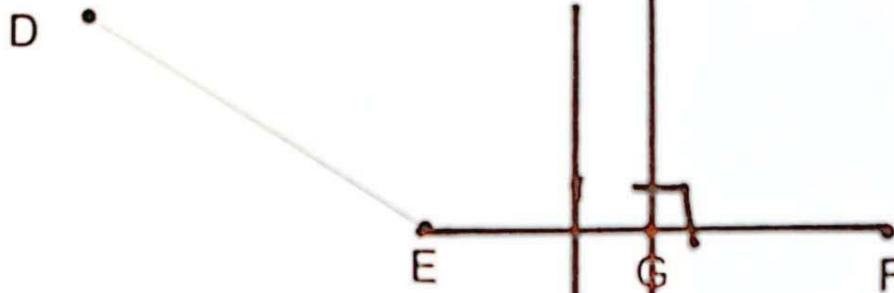


How many angles are the same? **8**

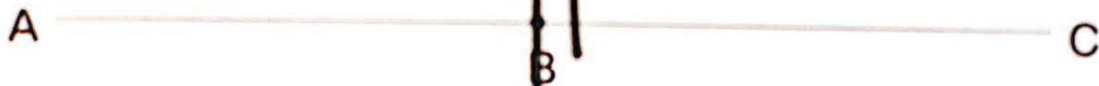
Right Angles

Right angles are made from a line that is perpendicular to a straight line.

Draw a line perpendicular to EF at point G:



Draw a line perpendicular to ABC at point B:



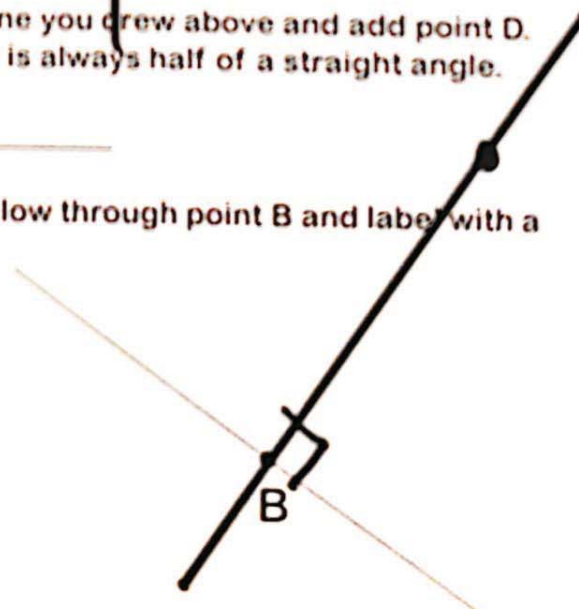
Pick a point on the perpendicular line you drew above and add point D. $\angle ABD$ is a *right angle*. A right angle is always half of a straight angle.

Measure $\angle ABD$: 90°

Draw a perpendicular to the line below through point B and label with a perpendicular symbol:



Perpendicular Symbol



All About Squares

A square is a quadrilateral with four equal sides and four right angles.

A

ABCD is a square.

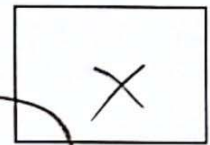
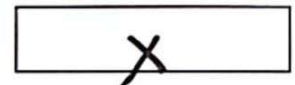
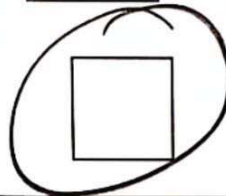
Compare the sides of the square. Are they all congruent?

Compare all of the angles. What do you notice?

D

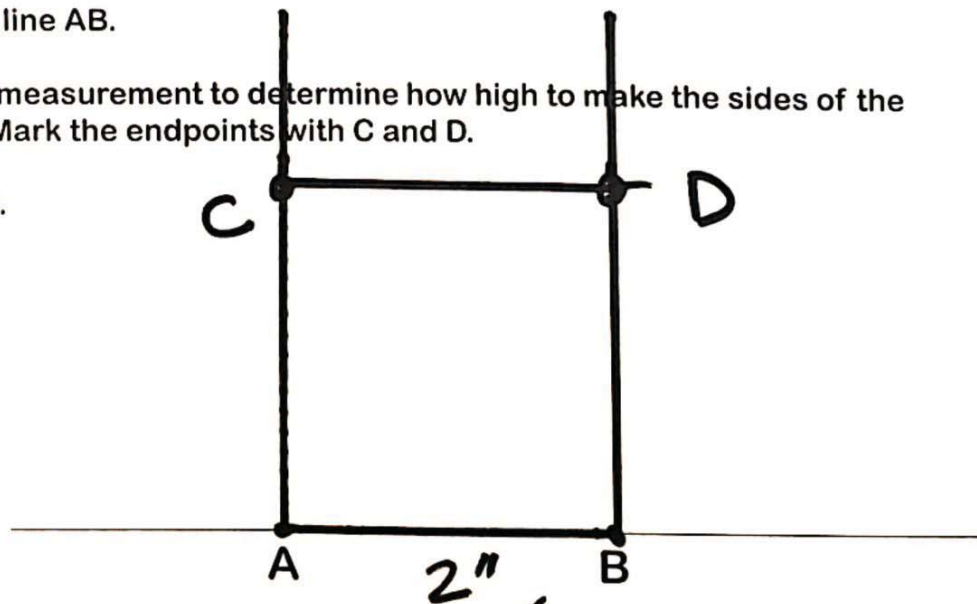
B

Which of the following quadrilaterals are squares? (circle):



Let's construct a square!

1. Use your protractor to mark two points, one above A and the other above point B. Line up the protractor's hole at A and make a mark at 90°. Do the same to mark a point above point B.
2. Draw a line perpendicular to A through the point above it. Do the same for B.
3. Measure line AB.
4. Use this measurement to determine how high to make the sides of the square. Mark the endpoints with C and D.
5. Draw CD.

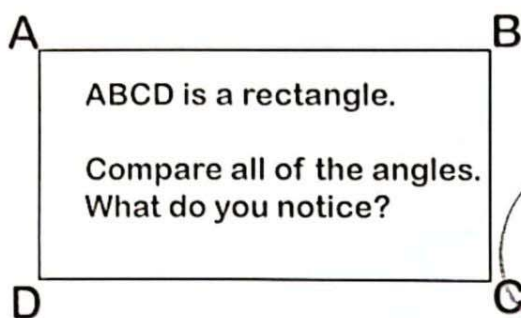


Check:

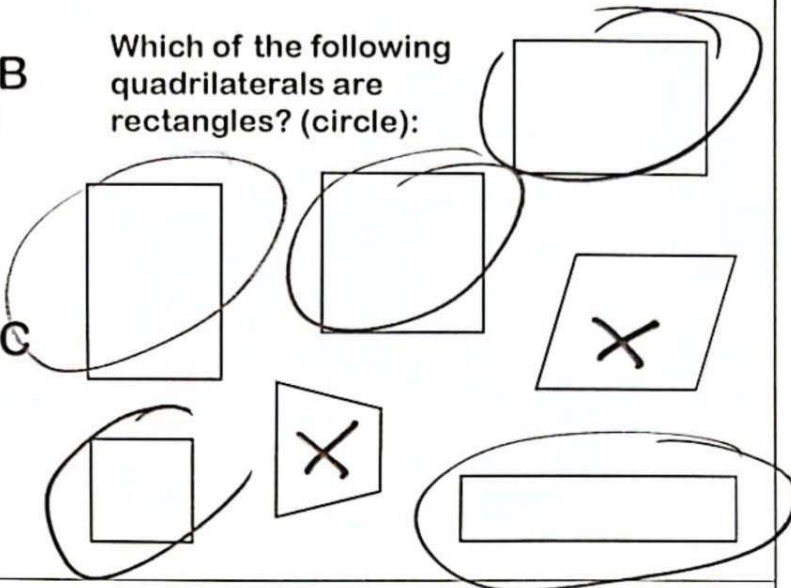
- are all four sides congruent? (use a ruler!) ✓
- are all four angles right angles? (use a protractor!) ✓

All About Rectangles

A rectangle is a quadrilateral with all right angles.

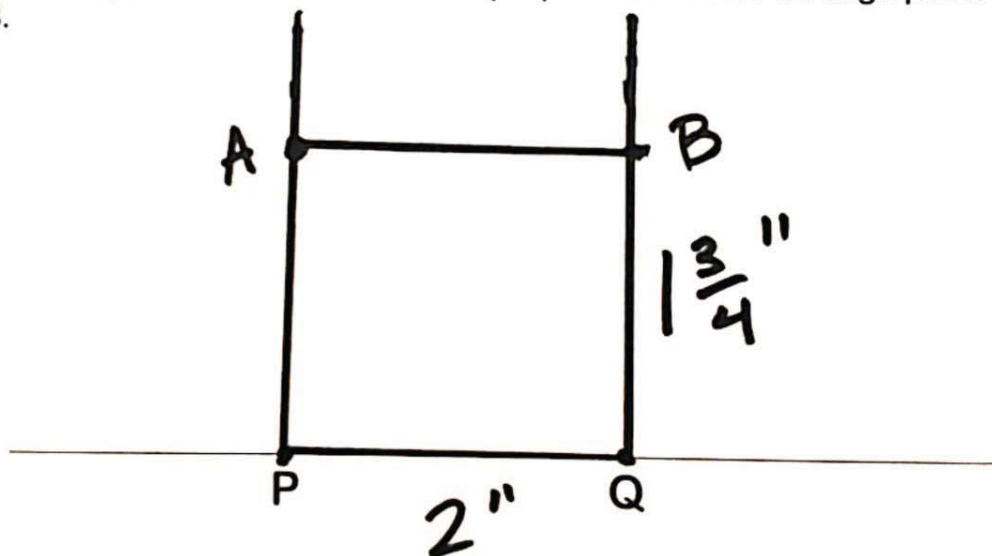


Which of the following quadrilaterals are rectangles? (circle):



Let's construct a rectangle!

1. Draw perpendiculars to segment \overline{PQ} through points P and Q.
2. Choose a point on the perpendicular through P and label it "A".
3. Measure \overline{PA} .
4. Use the line length \overline{PA} to mark "B" on the perpendicular line through point Q.
5. Draw \overline{AB} .



Check:

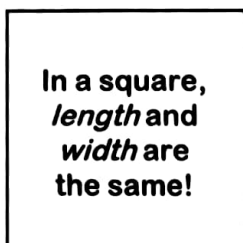
- are all four angles right angles? (use your protractor!)
- how do you know this object is a rectangle?

all right \angle 's ✓

Area of Rectangles and Squares

Area is the size of a surface of a two-dimensional figure.

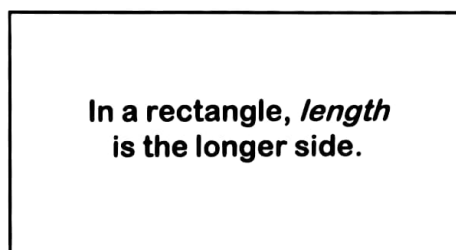
Think of area as the *square units* that a shape covers.



width

length

Area is the *length* times the *width* of a rectangle and a square.



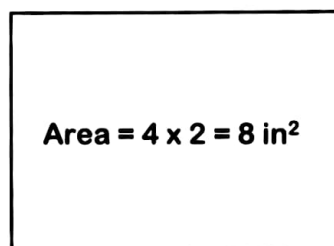
width

length

Area is measured in *square units*:

- square inches (in²)
- square feet (ft²)
- square cm (cm²)
- square meters (m²)
- square miles (mi²)
- square kilometers (km²)

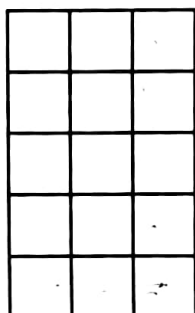
$$\text{Area} = L \times W$$



width = 2 inches

length = 4 inches

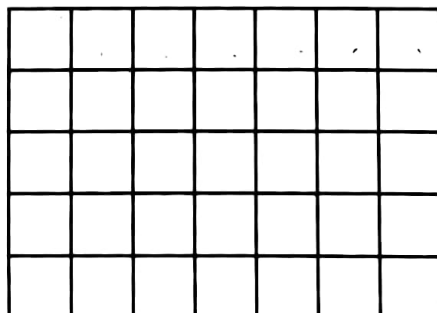
Count the number of squares to find the dimensions and area of each rectangle:



Length: 5

Width: 3

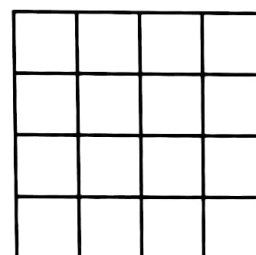
Area: 15 sq. units



Length: 7

Width: 5

Area: 35 sq. units



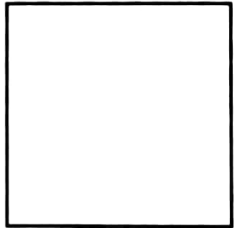
Length: 4

Width: 4

Area: 16 sq units

Area of Rectangles and Squares

Find the area of each shape using $\text{Area} = \text{Length} \times \text{Width}$



3 cm

3 cm

$$A = l \cdot w$$

Length: 3 cm

Width: 3 cm

Area: 9 cm^2

Don't forget units in your numbers!



1.2 cm

6 cm

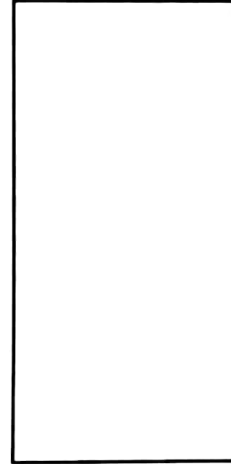
$$6 \times 1.2 = 7.2$$

Length: 6 cm

Width: 1.2 cm

Area: 7.2 cm^2

4 inches

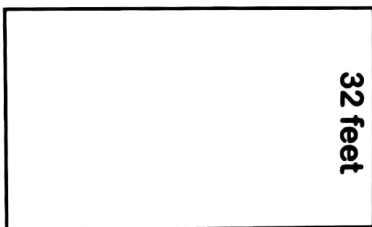


9 inches

Length: 9 in

Width: 4 in

Area: 36 in^2



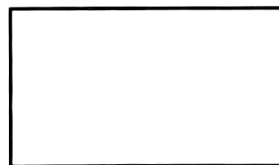
32 feet

75 feet

Length: 75 ft

Width: 32 ft

Area: 2400 ft^2



0.75 miles

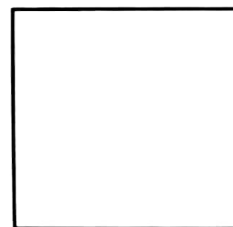
2.1 miles

miles = mi

Length: 2.1 mi

Width: 0.75 mi

Area: 1.575 mi^2



300 km

300 km

Length: 300 km

Width: 300 km

Area: $90,000 \text{ km}^2$

Area of Rectangles and Squares

Use your ruler to measure each shape and find the area using **Area = Length x Width**

Measure shapes in inches or cm to the nearest tenth

$1\frac{1}{4}"$

$1\frac{1}{4}"$

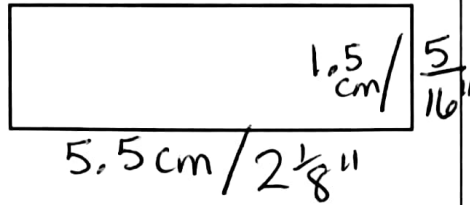
(or 3cm)

Length: 3cm / $1.25"$

Width: 3cm / $1.25"$

Area: 9cm^2 / 1.56in^2

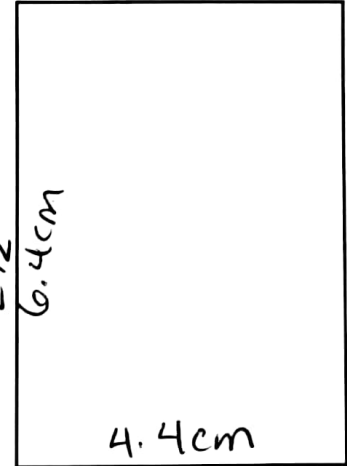
Don't forget units in your numbers!



Length: 5.5cm / $2\frac{1}{8}"$

Width: 1.5cm / $\frac{5}{16}"$

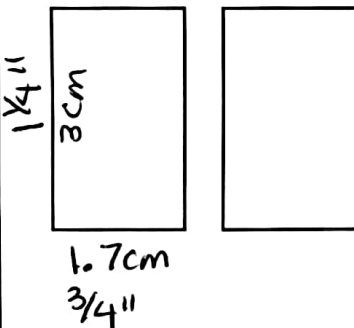
Area: 8.25cm^2 / 0.66in^2



Length: 4.4cm / $2\frac{1}{2}"$

Width: 4.4cm / $1\frac{3}{4}"$

Area: 28.2cm^2 / 4.4in^2

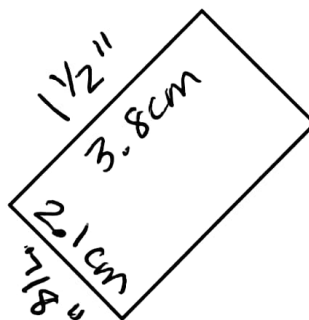


$$A = l \cdot w$$

Length: $1\frac{1}{4}"$ / 3cm

Width: $\frac{3}{4}"$ / 1.7cm

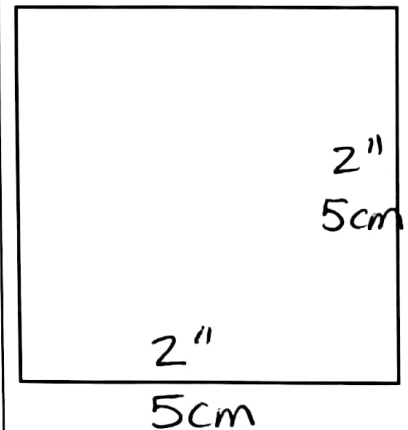
Total Area: 1.9in^2 / 10.2cm^2



Length: 3.8cm / $1\frac{1}{2}"$

Width: 2.1cm / $\frac{7}{8}"$

Area: 8cm^2 / 1.3in^2



Length: 2" / 5cm

Width: 2" / 5cm

Area: 4in^2 / 25cm^2

$$A = l \cdot w \Rightarrow \frac{A}{l} = w + \frac{A}{w} = l$$

Area of Rectangles and Squares

Using Area = Length x Width find the missing dimension(s) for each shape.

Area of
this
square is
25 cm²

$$A = 25 \text{ cm}^2$$

$$A = l \cdot w$$

$$l = w = 5 \text{ cm}$$

Don't forget units in your numbers!

Area is 25 cm²

10 ¼ cm

$$A = l \cdot w$$

$$25 = (10.25) w$$

$$\frac{25}{10.25} = \frac{10.25 w}{10.25}$$

$$w = \frac{25}{10.25} = 2.38 \text{ cm}$$

$$w = 2.38 \text{ cm}$$

6 ½ inches

Area = 64 in²

$$64 = (6.5)(w)$$

$$\frac{64}{6.5} = \frac{6.5 w}{6.5}$$

$$w = 9.9 \text{ in}$$

Area = 89 ft²

36 feet

$$A = \frac{89}{36} = \frac{36 \cdot w}{36}$$

$$w = 2.5 \text{ ft}$$

Area = 3.5 ft²

18 inches

$$18 \text{ in} \left(\frac{1 \text{ ft}}{12 \text{ in}} \right) = 1.5 \text{ ft}$$

$$3.5 = l (1.5)$$

$$\frac{3.5}{1.5} = \frac{l (1.5)}{1.5}$$

$$l = 2.3 \text{ ft}$$

1 foot = 12 inches

Area of square
is 6.4 mi²

w

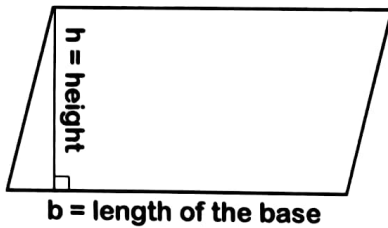
$$6.4 = l \cdot w$$

$$\sqrt{6.4} = \sqrt{l^2}$$

$$2.5 \text{ mi} = l$$

Area of a Parallelogram

A parallelogram is a simple quadrilateral with two sets of parallel sides.

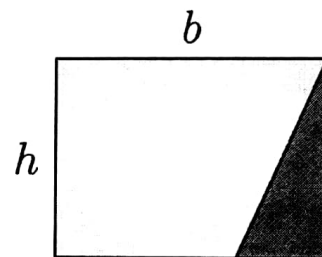
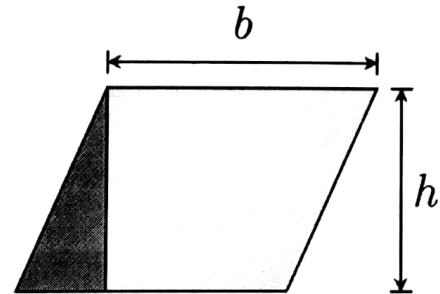
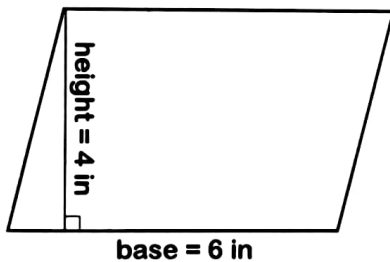


Area for a parallelogram is the height times the length of the base.

Make sure the height makes a 90° angle with the base.

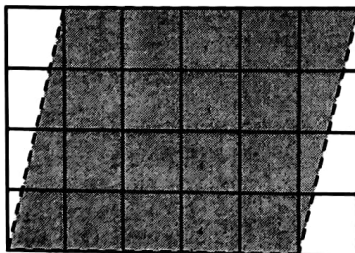
$$\text{Area} = b \times h$$

$$\text{Area} = 24 \text{ in}^2$$

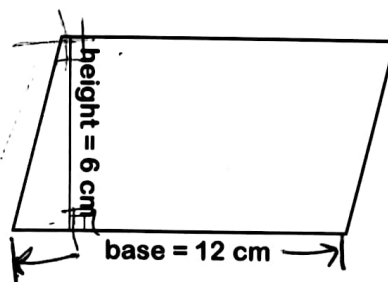


A parallelogram can be rearranged into a rectangle with the same area.

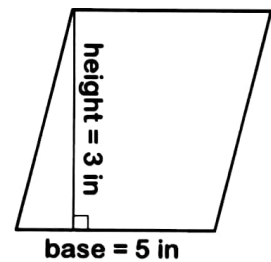
Find the dimensions and area of each parallelogram:



Base: 5 units
Height: 4 units
Area: 20 sq. units



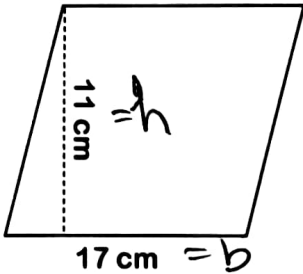
Base: 12 cm
Height: 6 cm
Area: 72 cm^2



Base: 5 in
Height: 3 in
Area: 15 in^2

Area of Parallelograms

Find the area of each shape using **Area = base x height**



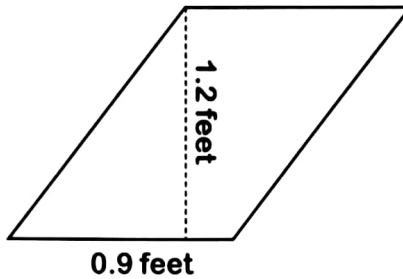
$$A = b \cdot h$$

Base: 17 cm

Height: 11 cm

Area: 187 cm²

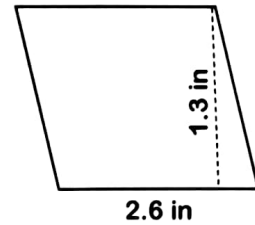
Don't forget units in your numbers!



Base: 0.9'

Height: 1.2'

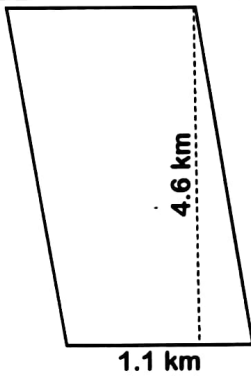
Area: 1.08 sq. ft



Base: 2.6"

Height: 1.3"

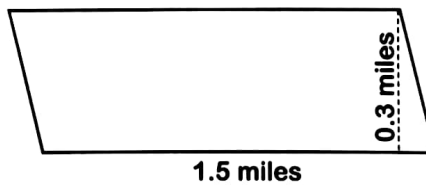
Area: 3.38 in²



Base: 1.1 km

Height: 4.6 km

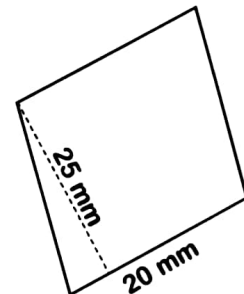
Area: 5.06 km²



Base: 1.5 mi

Height: 0.3 mi

Area: 0.45 mi²



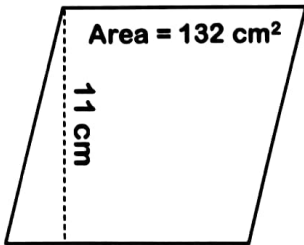
Base: 20 mm

Height: 25 mm

Area: 500 mm²

Area of Parallelograms

Find the area of each shape using Area = base x height



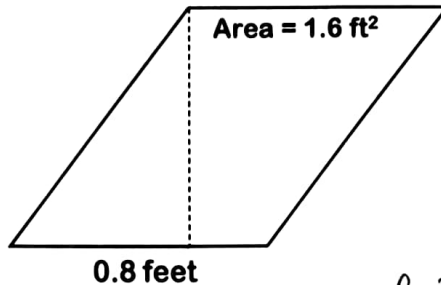
$$b = \frac{A}{h} = \frac{132 \text{ cm}^2}{11 \text{ cm}}$$

Base: 12 cm

Height: 11 cm

Area: 132 cm²

Don't forget units in your numbers!

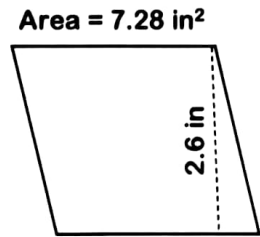


$$h = \frac{A}{b} = \frac{1.6 \text{ ft}^2}{0.8 \text{ ft}}$$

Base: 0.8 ft

Height: 2 ft

Area: 1.6 ft²

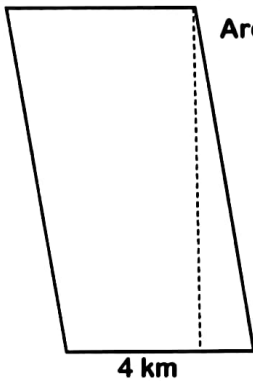


$$b = \frac{A}{h} = \frac{7.28 \text{ in}^2}{2.6 \text{ in}}$$

Base: 2.8 in

Height: 2.6 in

Area: 7.28 in²

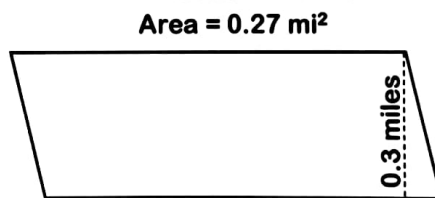


$$h = \frac{46}{4}$$

Base: 4 km

Height: 11.5 km

Area: 46 km²

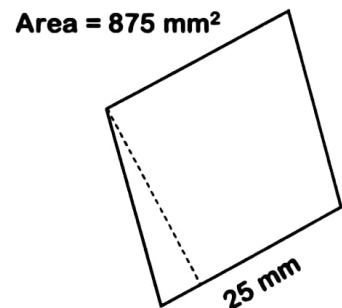


$$b = \frac{0.27}{0.3}$$

Base: 0.9 mi

Height: 0.3 mi

Area: 0.27 mi²



$$h = \frac{875}{25}$$

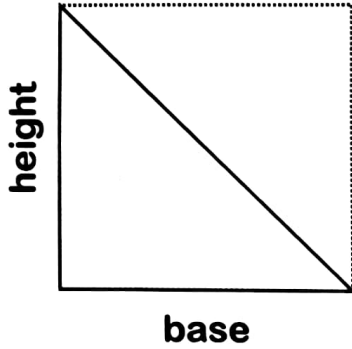
Base: 25 mm

Height: 35 mm

Area: 875 mm²

Area of Triangles

Think of the area of a triangle as *half* the area of a square or rectangle.

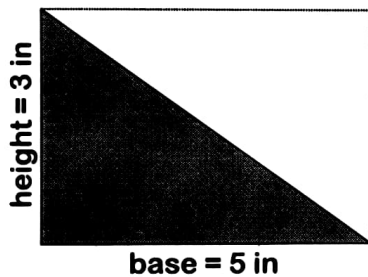


Area for a triangle is one-half of the two sides multiplied together.

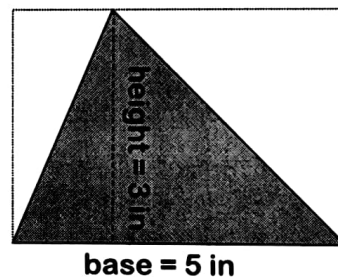
The area is $\frac{1}{2}$ times the *base* times the *height* of a triangle.

$$\text{Area} = \frac{1}{2} b \times h$$

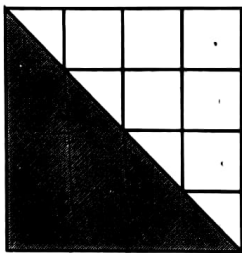
$$\text{Area} = \frac{1}{2} (5 \times 3) = 4 \text{ in}^2$$



$$\text{Area} = \frac{1}{2} (5 \times 3) = 4 \text{ in}^2$$



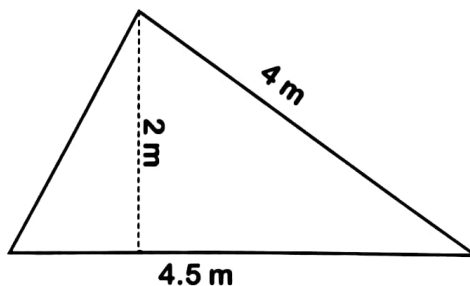
Find the dimensions and area of each triangle:



Base: 4 units

Height: 4 units

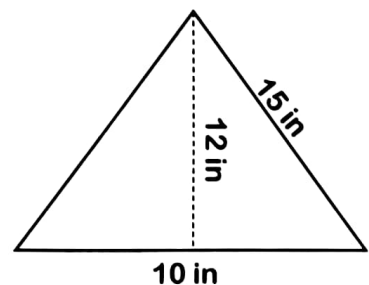
Area: $\frac{1}{2}(4)(4) = 8 \text{ sq. units}$



Base: 4.5 m

Height: 2 m

Area: $\frac{1}{2}(4.5)(2) = 4.5 \text{ m}^2$



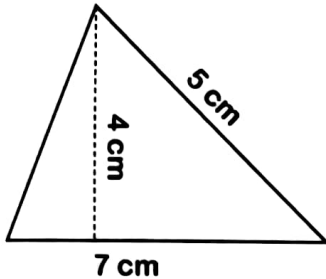
Base: 10"

Height: 12"

Area: $\frac{1}{2}(10)(12) = 60 \text{ sq. in}$

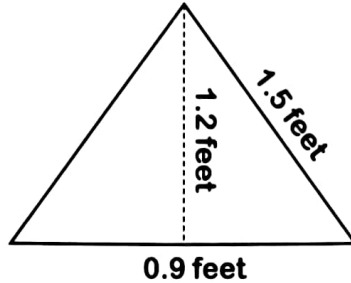
Area of Triangles

Find the area of each shape using $\text{Area} = \frac{1}{2} \text{ base} \times \text{height}$

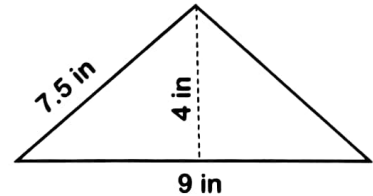


Base: 7 cm
Height: 4 cm
Area: $\frac{1}{2}(7)(4) = 14 \text{ cm}^2$

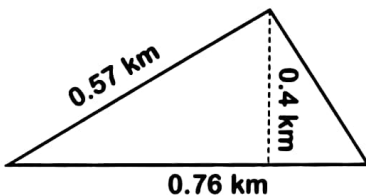
Don't forget units in your numbers!



Base: 0.9'
Height: 1.2'
Area: $\frac{1}{2}(0.9)(1.2) = 0.54 \text{ in}^2$

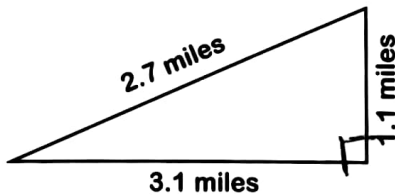


Base: 9 in
Height: 4 in
Area: $\frac{1}{2}(4)(9) = 18 \text{ in}^2$



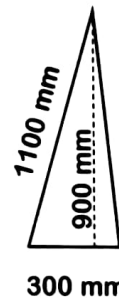
$$A = 0.152 \text{ km}^2$$

Base: 0.76 km
Height: 0.4 km
Area: $\frac{1}{2}(0.76)(0.4)$



$$A = 1.705 \text{ mi}^2$$

Base: 3.1 miles
Height: 1.1 miles
Area: $(\frac{1}{2})(3.1)(1.1)$

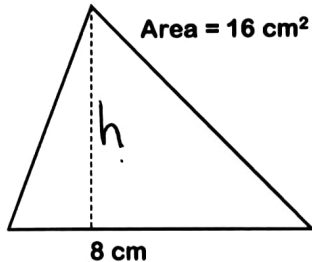
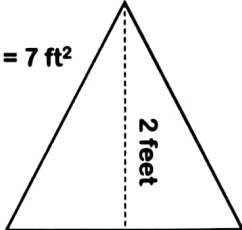
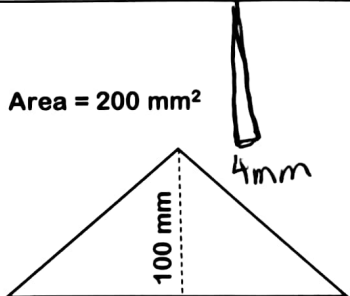
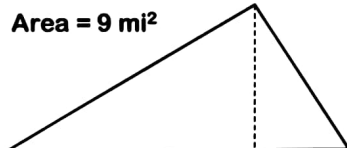
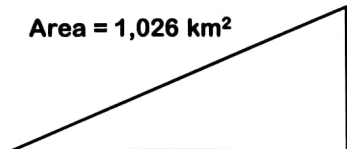
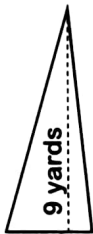


$$A = 135,000 \text{ mm}^2$$

Base: (300 mm)
Height: 900 mm
Area: $\frac{1}{2}(300)(100)$

Area of Triangles

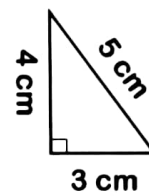
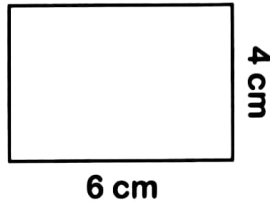
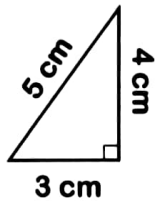
Using Area = $\frac{1}{2}$ Base x Height, find the missing dimension(s) for each shape.

 <p>Area = 16 cm²</p> <p>8 cm</p> $h = \frac{2A}{b} = \frac{2(16)}{8}$ <p>Base: 8 cm Height: 4 cm Area: 16 cm²</p>	<p>Don't forget units in your numbers!</p>  <p>Area = 7 ft²</p> <p>2 feet</p> $b = \frac{2A}{h} = \frac{2(7)}{2}$ <p>Base: 7 ft Height: 2 ft Area: 7 ft²</p>	 <p>Area = 200 mm²</p> <p>100 mm</p> <p>4mm</p> $b = \frac{2A}{h} = \frac{2(200)}{100}$ <p>Base: 4 mm Height: 100 mm Area: 200 mm²</p>
<p>Area = 9 mi²</p>  <p>6 miles</p> $h = \frac{2A}{b} = \frac{2(9)}{6}$ <p>Base: 6 mi Height: 3 mi Area: 9 mi²</p>	<p>Area = 1,026 km²</p>  <p>76 km</p> $h = \frac{2A}{b} = \frac{2(1026)}{76}$ <p>Base: 76 km Height: 27 km Area: 1,026 km²</p>	<p>Area = 13.5 yd²</p>  <p>9 yards</p> $b = \frac{2A}{h} = \frac{2(13.5)}{9}$ <p>Base: 3 yd Height: 9 yd Area: 13.5 yd²</p>

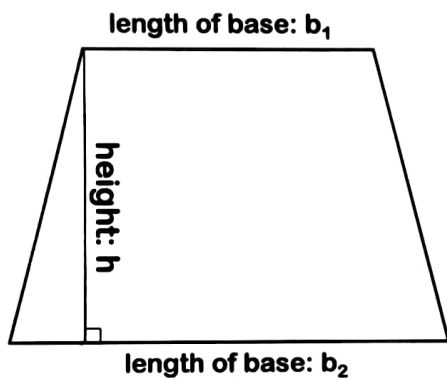
Area of Trapezoids

A trapezoid is one or two triangles and a rectangle (or square) combined.

Three figures are shown below. Find the area of each.



Area:



Rectangle Area = $b \times h$

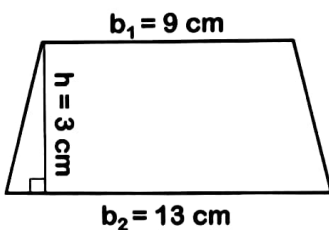
Triangle Area = $\frac{1}{2} b \times h$

To find the area of a trapezoid, we combine the two areas above into this:

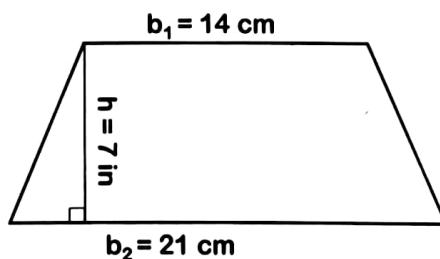
Trapezoid Area = $\frac{1}{2} (b_1 + b_2) h$

Remember! Units for area are *squared* like this: cm^2 or in^2

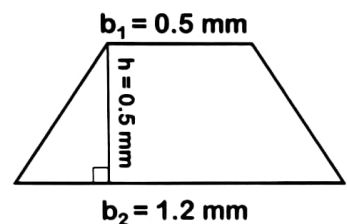
Find the dimensions and area of each trapezoid:



$$\begin{aligned} \text{Area: } & \frac{1}{2} (b_1 + b_2) h \\ & = \frac{1}{2} (9 + 13) (3) \\ & = \frac{1}{2} (22) (3) \\ & = \boxed{33 \text{ cm}^2} \end{aligned}$$



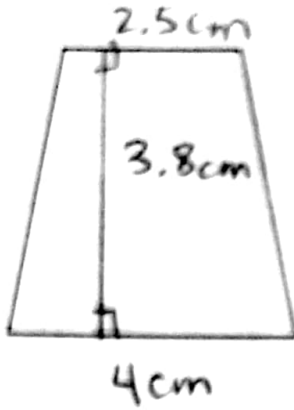
$$\begin{aligned} \text{Area: } & \frac{1}{2} (14 + 21) (7) \\ A = & \frac{1}{2} (35) (7) \\ \boxed{A = 122.5 \text{ cm}^2} \end{aligned}$$



$$\begin{aligned} \text{Area: } & \frac{1}{2} (0.5 + 1.2) (0.5) \\ \boxed{A = 0.425 \text{ mm}^2} \end{aligned}$$

Area of Trapezoids: $A = \frac{1}{2} (b_1 + b_2) h$

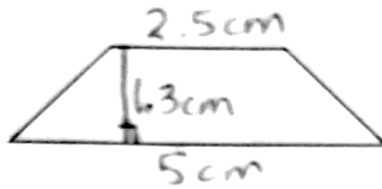
Use your ruler to measure each shape and find the area (in cm^2)



$$A = \frac{1}{2} (2.5 + 4) (3.8)$$

$$A = \frac{1}{2} (6.5) (3.8)$$

$$\text{Area: } 12.35 \text{ cm}^2$$

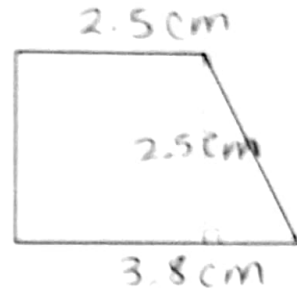


$$A = \frac{1}{2} (2.5 + 5) (1.3)$$

$$A = \frac{1}{2} (7.5) (1.3)$$

$$A = 4.875 \text{ cm}^2$$

Area:

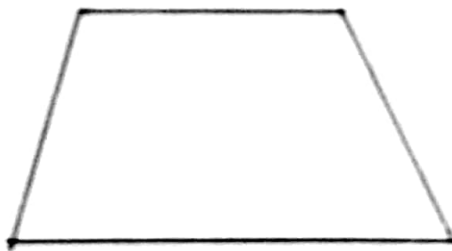


$$A = \frac{1}{2} (2.5 + 3.8) (2.5)$$

$$A = 7.875 \text{ cm}^2$$

Area:

Draw one trapezoid in each space below. Measure the sides you need and calculate the area.



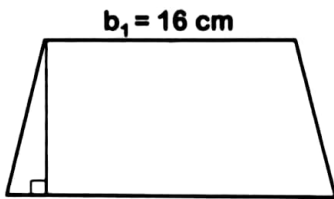
Area:

Area:

Area:

Area of Trapezoids: $A = \frac{1}{2} (b_1 + b_2) h$

These trapezoids are missing a dimension! Can you figure out each one?
Hint: You may have to re-arrange the trapezoid area equation to use it to find the missing side.



Area = 252 cm²

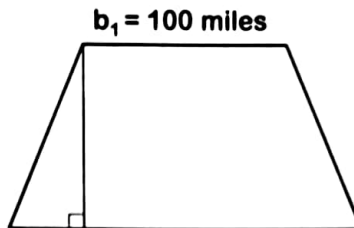
Base 1: 16 cm

Base 2: 26 cm

Height: $h = 12$ cm

Formula: $h = \frac{2A}{b_1 + b_2}$

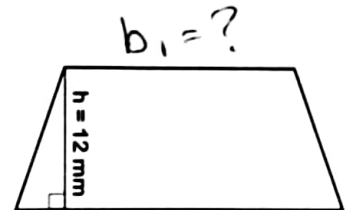
Area: 252 cm²



Area = 5,200 mi²

$$h = \frac{2A}{(b_1 + b_2)} = \frac{2(5200)}{(160 + 100)}$$

$$h = 40 \text{ miles}$$



b₂ = 22 mm

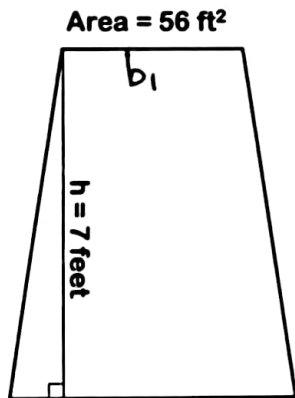
Area = 204 mm²

$$b_1 = \frac{2A}{h} - b_2$$

$$b_1 = \frac{2(204)}{(12)} - 22$$

$$b_1 = 34 - 22$$

$$b_1 = 12 \text{ mm}$$



Area = 56 ft²

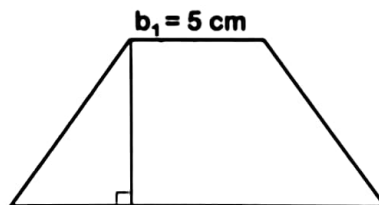
b₂ = 8 feet

$$b_1 = \frac{2A}{h} - b_2$$

$$b_1 = \frac{2(56)}{7} - 8$$

$$b_1 = 16 - 8$$

$$b_1 = 8$$



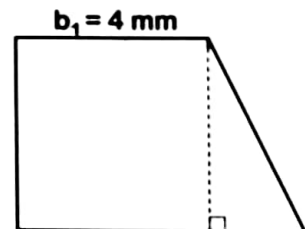
b₂ = 15 cm

Area = 20 cm²

$$h = \frac{2A}{(b_1 + b_2)}$$

$$h = \frac{2(20 \text{ cm}^2)}{(5 + 15) \text{ cm}}$$

$$h = 2 \text{ cm}$$



b₂ = 7 mm

Area = 22 mm²

$$h = \frac{2A}{(b_1 + b_2)}$$

$$h = \frac{2(22)}{(4 + 7)}$$

$$h = \frac{2(22)}{11}$$

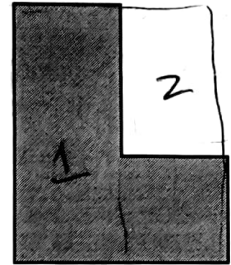
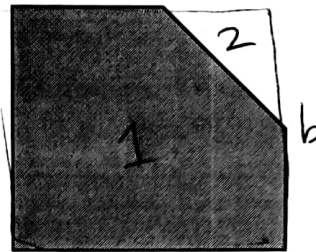
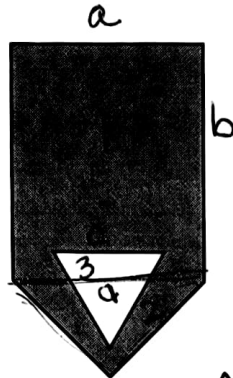
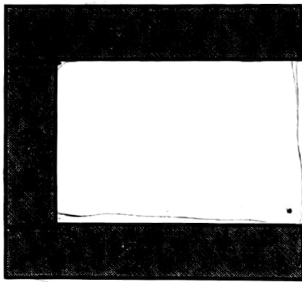
$$h = 4 \text{ mm}$$

Area of Composite Figures

A *composite figure* is made up of two or more shapes.

How would you decompose the figures below?

Discuss and label the different ways could break each apart into familiar shapes.



$$A = A_1 + A_2 + A_3$$

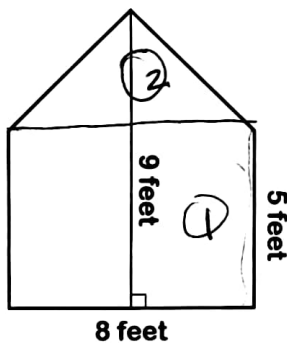
$$A = bh_1 + bh_2 + bh_3$$

$$\boxed{1} + \boxed{2} - \boxed{3}$$

$$a \boxed{1} - \boxed{2}$$

$$\boxed{1} - \boxed{2}$$

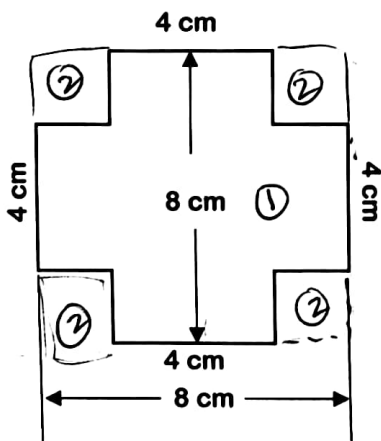
To find the area of a composite shape, first break the figure into familiar shapes, then add or subtract the area for each shape as appropriate.



$$\boxed{1} \quad 8' \quad 5' \quad A_R = 8 \times 5 = 40 \text{ ft}^2$$

$$\boxed{2} \quad \triangle \quad 8' \quad 4' \quad A_T = \frac{1}{2}(8)(4) = 16 \text{ ft}^2$$

$$A_{\text{total}} = A_R + A_T = 40 + 16 = \boxed{56 \text{ ft}^2}$$



$$\boxed{1} \quad 8 \quad A_1 = 8 \cdot 8 = 64 \text{ cm}^2$$

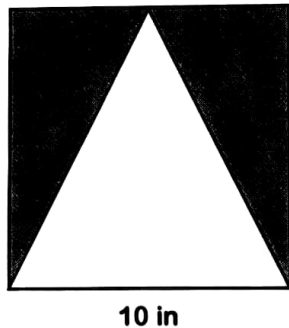
$$- 4 \quad \boxed{2} \quad 2 \Rightarrow A_2 = (2)(2) = 4 \text{ cm}^2$$

$$4(4 \text{ cm}^2) = 16 \text{ cm}^2$$

$$64 - 16 \Rightarrow \text{Total Area} = \boxed{48 \text{ cm}^2}$$

Area of Composite Figures

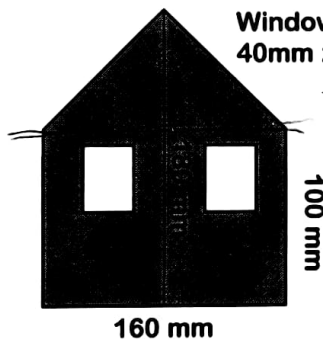
Find the area of the shaded regions by decomposing it into familiar shapes.



$$\textcircled{1} \begin{array}{|c|} \hline 10 \\ \hline \end{array} \quad A_1 = b \cdot h = (10)(10) = 100 \text{ in}^2$$

$$- \textcircled{2} \quad A_2 = \frac{1}{2}bh = \frac{1}{2}(10)(10) = 50 \text{ in}^2$$

$$A_{\text{Total}} = A_1 - A_2 = 100 - 50 = \boxed{50 \text{ in}^2}$$



Windows are each
40mm x 20 mm

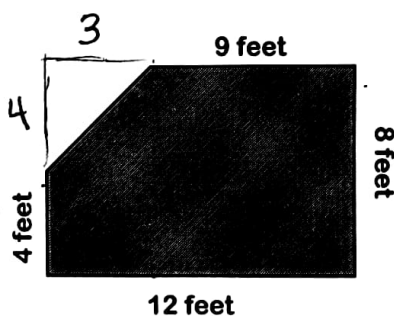
$$+ \textcircled{1} \begin{array}{|c|} \hline 100 \\ \hline \end{array} \quad A_1 = b \cdot h = (160)(100) = 16,000 \text{ mm}^2$$

$$+ \textcircled{2} \quad A_2 = \frac{1}{2}bh = \frac{1}{2}(160)(80) = 6,400 \text{ mm}^2$$

$$- 2 \textcircled{3} \begin{array}{|c|} \hline 40 \\ \hline \end{array} \quad A_3 = bh = (20)(40) = 800 \text{ mm}^2$$

$$\boxed{A_{\text{Total}} = 20,800 \text{ mm}^2}$$

$$A_{\text{Total}} = A_1 + A_2 - 2A_3 = 16,000 + 6,400 - 2(800) =$$



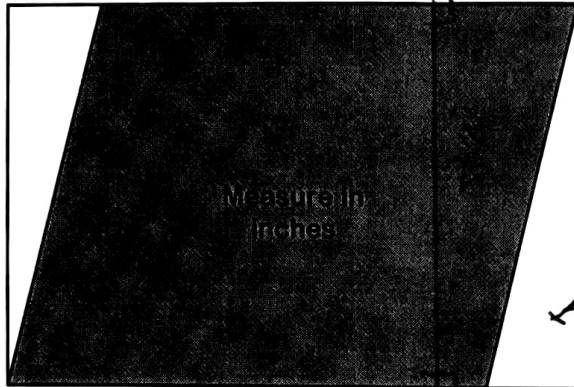
$$\textcircled{1} \quad A = b \cdot h = (12)(8) = 96 \text{ ft}^2$$

$$\textcircled{2} \quad A = \frac{1}{2}bh = \frac{1}{2}(4)(3) = 6 \text{ ft}^2$$

$$A_{\text{Total}} = 96 - 6 = \boxed{90 \text{ ft}^2}$$

Area of Composite Figures

Calculate the area of the shaded regions below.
Use your ruler to measure any information you need.



2.5"

$$A = b \cdot h$$

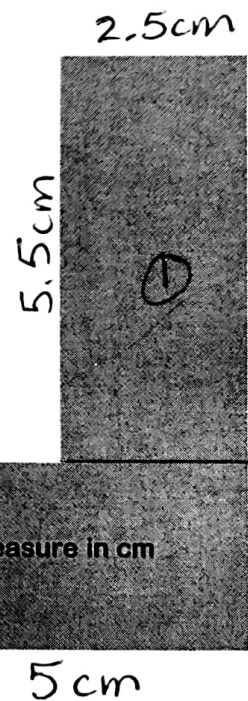
$$A = (2.5)(2)$$

$$A = 5 \text{ in}^2$$

$$A_1 = b \cdot h = (2.5)(5.5) = 13.75 \text{ cm}^2$$

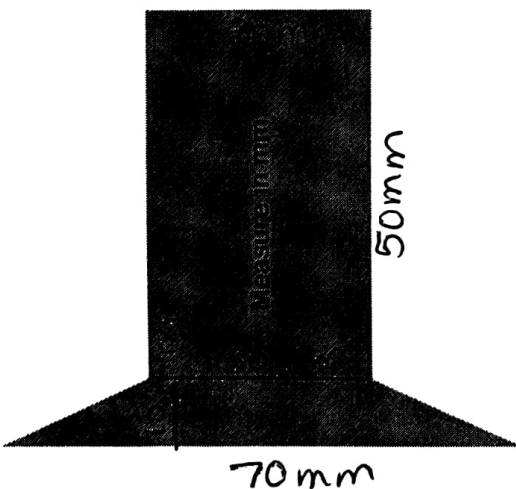
$$A_2 = (5)(2.5) = 12.5 \text{ cm}^2$$

$$A_{\text{Total}} = A_1 + A_2 = 26.25 \text{ cm}^2$$



$$\textcircled{1} = b \cdot h$$

$$\textcircled{2} = \frac{1}{2}(b_1 + b_2)h$$



70 mm

$$A_1 = (30)(50) = 1500 \text{ mm}^2$$

$$A_2 = \frac{1}{2}(30 + 70)(10)$$

$$= \frac{1}{2}(100)(10) = 500 \text{ mm}^2$$

$$A_{\text{Total}} = 1500 + 500 = 2,000 \text{ mm}^2$$

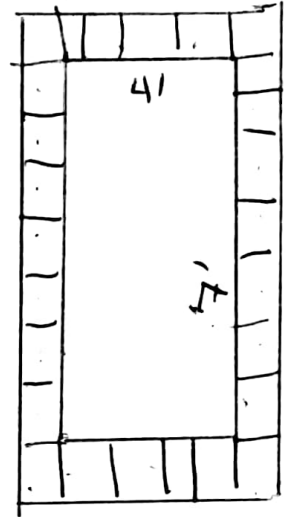
Area of Composite Figures

Solve the word problems by first drawing the shapes.

Tony is placing new 12" square tiles around his 7 ft x 4' pool. Tiles cost \$2.25 each. How much is this project going to cost? (Hint: Don't forget the corners!)

$$\begin{aligned} \text{Total \# tiles} &= 7(2) + 4(2) + 4 \text{ (corners)} \\ &= 14 + 8 + 4 = 26 \text{ tiles} \end{aligned}$$

$$\frac{\$2.25}{\text{tile}} (26 \text{ tiles}) = \boxed{\$58.50}$$



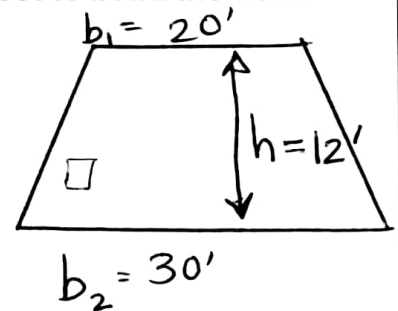
Sophia is building a new deck for her home. The deck is a trapezoid shape with parallel sides measuring 20 feet and 30 feet, and distance between the parallel sides is 12 feet. If wood is \$3.40 per sq ft, how much will it cost to build the deck?

$$A = \frac{1}{2}(b_1 + b_2)h$$

$$A = \frac{1}{2}(20 + 30)(12)$$

$$A = \frac{1}{2}(50)(12) = 300 \text{ sq ft}$$

$$\frac{\$3.40}{\text{sq. ft}} (300 \text{ sq ft}) = \boxed{\$1,020}$$



Area of Composite Figures

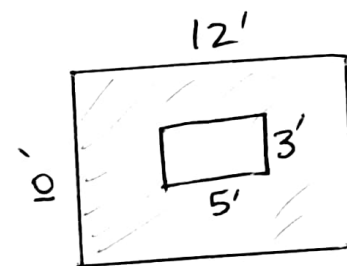
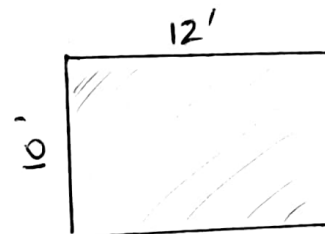
Solve the word problems by first drawing the shapes.

Arthur lives in a home with ten foot ceilings. He is going to paint two 12-foot long walls with two coats of paint. One wall has a 3 ft x 5 ft window. One gallon of paint covers 400 sq ft. How much paint should he buy?

$$A = b \cdot h \Rightarrow A = (10)(12) + (10)(12) - (5)(3)$$

$$A = \underset{\substack{\uparrow \text{wall 1} \quad \uparrow \text{wall 2} \quad \uparrow \text{window}}}{120 + 120 - 15} = \underset{\substack{\uparrow \text{total}}}{225} \text{ sq ft}$$

$$(225 \text{ sq ft})(2 \text{ coats of paint}) = \underline{450 \text{ sq ft}} \rightarrow \boxed{2 \text{ gallons}}$$



Trent is laying new tile in his kitchen. The kitchen measures 14 feet by 20 feet. The kitchen has cabinets that extend 2 feet into the room sitting along the 14 foot side. There's also a 6 x 8 foot center island that he is going to work around. The tiles are 12 inches square for \$5.50. How much tile does he need and how much will it cost?

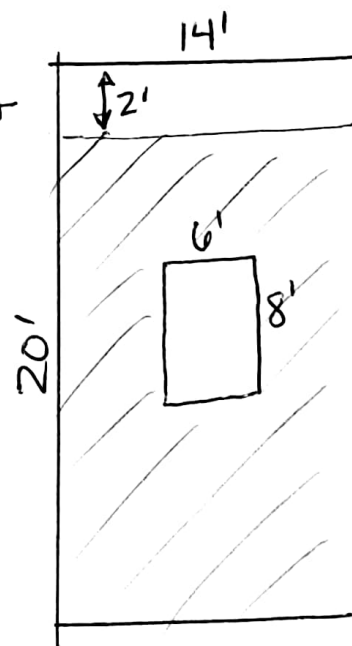
$$A = bh: A = (18)(14) - (6)(8)$$

$$A = 252 - 48 = \underline{204 \text{ sq ft}}$$

$$\begin{array}{|c|} \hline 1 \text{ ft} \\ \hline \$5.50 \\ \hline 1 \text{ ft} \\ \hline \end{array}$$

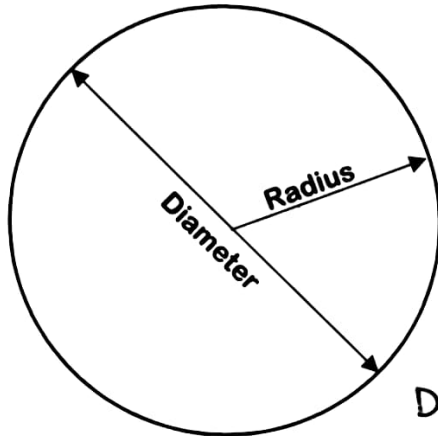
204 tiles

$$204(\$5.50) = \boxed{\$1,122}$$



Area of Circles

Circles are round shapes without any corners or line segments.



The Radius (r)

This is measured *from* the center of a circle to any point *on* the circle.

$$\text{Area} = \pi r^2$$

The Diameter (d)

Any straight line passing through the *center* of the circle with endpoints *on* the circle.

$$\text{Area} = \frac{1}{4} \pi d^2$$

$$\pi = 3.14159\dots$$

π : (*pi*, pronounced "pie") is a number a little larger than 3.

π is the number we get when we divide the circumference by the diameter.

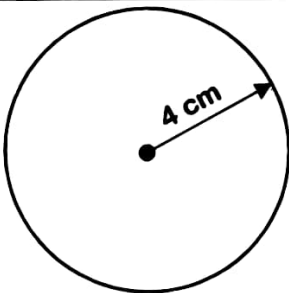
We will use $\pi = 3.14$ for our calculations.

The Circumference (C)

This is the distance measured *around* the entire circle.

$$C = 2 \pi r = \pi d$$

Find the dimensions and area of each circle:

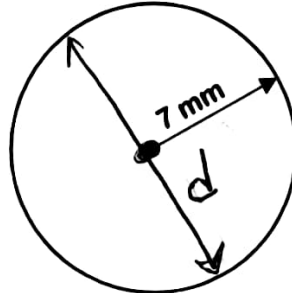


$$r = 4 \text{ cm}$$

$$d = 2 \times 4 \text{ cm} = 8 \text{ cm}$$

$$C = 2 \pi r = 2 (3.14)(4 \text{ cm}) = 25.1 \text{ cm}$$

$$\text{Area} = \pi r^2 = (3.14)(4 \text{ cm})^2 = 50.3 \text{ cm}^2$$

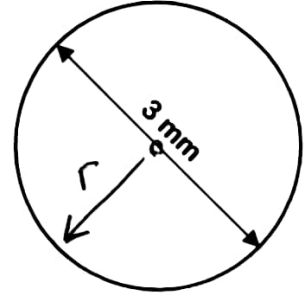


$$r: 7 \text{ mm}$$

$$d: 14 \text{ mm}$$

$$C: 2(\pi)(7 \text{ mm}) = 43.96 \text{ mm}$$

$$\text{Area: } \pi r^2 = \pi (7 \text{ mm})^2 = 153.86 \text{ mm}^2$$



$$r: \frac{1}{2}(3 \text{ mm}) = 1.5 \text{ mm}$$

$$d: 3 \text{ mm}$$

$$C: \pi d = \pi (3 \text{ mm}) = 9.42 \text{ mm}$$

$$\text{Area: } \pi r^2 = \pi (1.5 \text{ mm})^2 \sim 7 \text{ mm}^2$$

Area of Circles

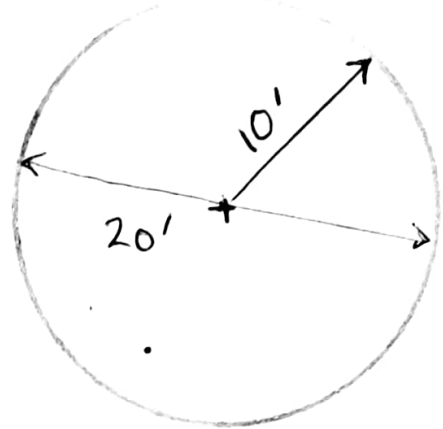
$$\text{Circumference} = 2\pi r$$

$$\text{Area} = \pi r^2 = \frac{1}{4}\pi d^2$$

$$\text{Use: } \pi = 3.14$$

A wooden horse on a carousel travels in a circle. The carousel is 20 feet in diameter. How far does the horse travel each time it completes a circle?

$$\begin{aligned} C &= 2\pi r \\ C &= 2(3.14)(10') \\ C &= 62.8 \text{ feet} \end{aligned}$$



What is both the radius and diameter of the carousel?

$$\begin{aligned} r &= 10' \\ d &= 20' \end{aligned}$$

The owner needs to install a new floor in the carousel. The carousel has a 4 foot hole in the center for the machinery that turns the carousel. How much wood does he need, in square feet?

$$A_1 = \pi (10')^2 = 314 \text{ ft}^2$$

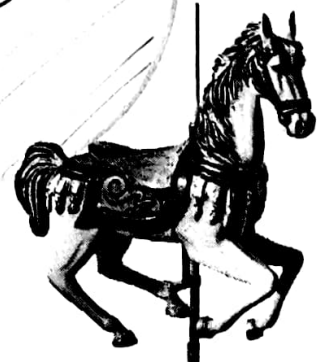
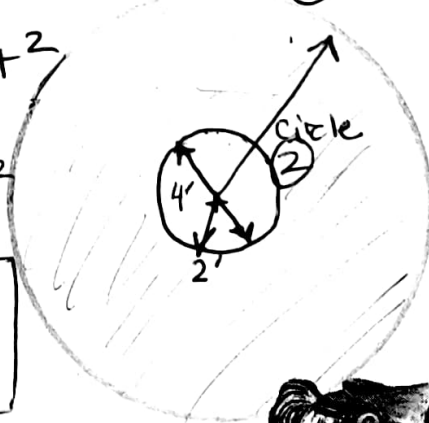
$$A_2 = \pi (2')^2 = 12.56 \text{ ft}^2$$

$$A_{\text{TOTAL}} = 314 - 12.56 \text{ ft}^2$$

$$A_{\text{TOTAL}} = 301.44 \text{ ft}^2$$

$$A_{\text{TOTAL}} = A_1 - A_2$$

Circle ①



Area of Circles

$$\text{Circumference} = 2\pi r$$

$$\text{Area} = \pi r^2 = \frac{1}{4}\pi d^2$$

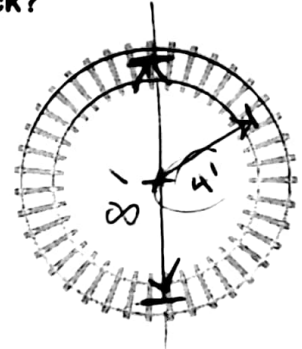
$$\text{Use: } \pi = 3.14$$

An electric model train travels in a circular loop of track made from 4-foot radius arcs. How far does the train travel after 10 laps around the track?

$$r = 4' \quad d = 8'$$

$$C = 2\pi r = 2(3.14)(4') = 25.12'$$

$$10 \text{ Laps} = 10(25.12') = \boxed{251.2'}$$



What is both the radius and diameter of the track?

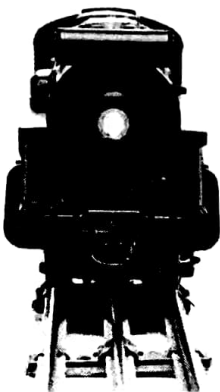
$$d = 8' \quad r = 4'$$

Approximately how many laps does the train need to do to travel 500 meters?

$$3.28 \text{ ft} = 1 \text{ m}$$

$$25.12 \text{ ft} \left(\frac{1 \text{ m}}{3.28 \text{ ft}} \right) = 7.7 \text{ m for 1 lap}$$

$$\frac{500 \text{ m}}{7.7 \text{ m/lap}} = \boxed{65.3 \text{ Laps}}$$



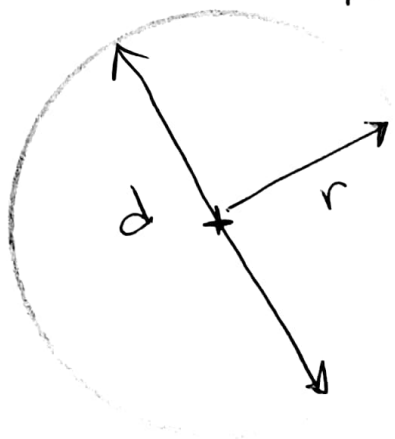
Area of Circles

Circumference = $2\pi r$

Area = $\pi r^2 = \frac{1}{4}\pi d^2$

Use: $\pi = 3.14$

A farmer's rotating irrigation sprinkler setup covers an area of 2800 meters.²
About how big is the circle that it covers?



$$A = 2800 \text{ m}^2 \quad \frac{A}{\pi} = \frac{\pi r^2}{\pi} \Rightarrow r^2 = \frac{A}{\pi}$$

$$\text{so } r = \sqrt{\frac{A}{\pi}}$$

$$r = \sqrt{\frac{2800 \text{ m}^2}{3.14}} = \boxed{29.9 \text{ m}}$$

$$d = 2 \cdot r = 2(29.9)$$

$$\boxed{d = 59.7 \text{ m}}$$

A horse trainer wants to create a 50-foot diameter bullpen (a round fenced-in enclosure) for training and lunging horses. How many fences does she need if they come in 7 foot sections?

$$C = \pi d = (3.14)(50')$$

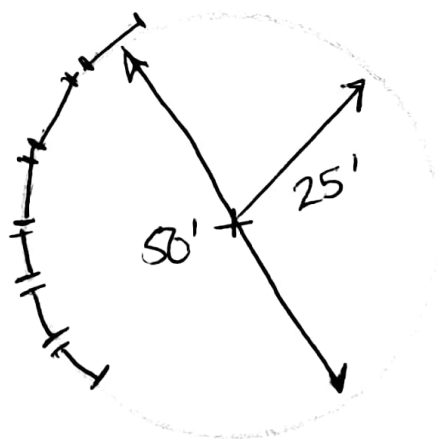
$$C = 157 \text{ ft}$$

$$\text{Total \# of fences} = \frac{157 \text{ ft}}{7 \text{ ft/fence}}$$

$$\rightarrow = \boxed{23 \text{ fences}}$$

$$C = \pi d$$

$$C = 2\pi r$$



Practice Test

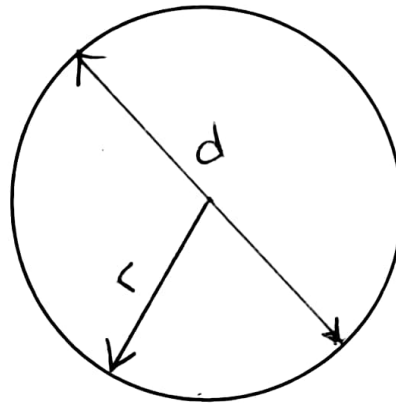
Use your ruler (use cm) to determine the dimensions, and then calculate the perimeter and area of each shape below. Round to the nearest tenth of a cm.

Diameter: 5.2 cm

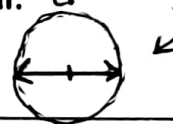
Radius: 2.6 cm

Circumference: $\pi d = 16.3 \text{ cm}$

Area: $\pi r^2 = \pi (2.6 \text{ cm})^2$
 $= 21.2 \text{ cm}^2$



Now draw a similar circle that is one quarter the size of the original. $d = 1.3 \text{ cm}$



Length: 3.7 cm

Width: 1.5 cm

Perimeter: $2(3.7) + 2(1.5) = 10.4 \text{ cm}$

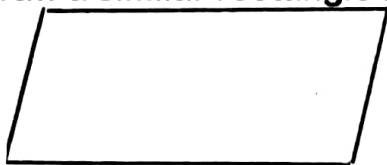
Area: $= b \cdot h = (3.7)(1.5)$
 $= 5.5 \text{ cm}^2$



3.7 cm

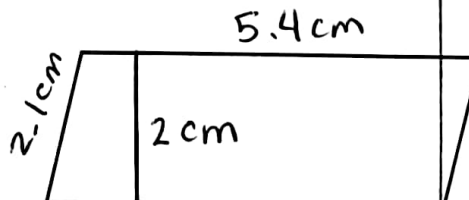
1.5 cm

Now draw a similar rectangle that is three times the size of the original.



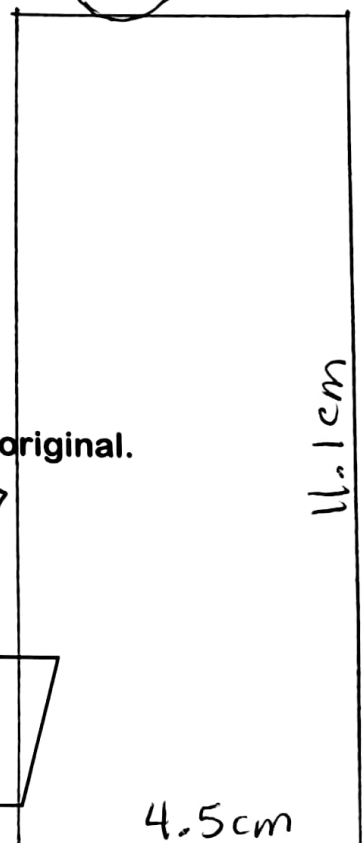
Length: 5.4 cm

height: 2 cm



Perimeter: $2(2.1 \text{ cm}) + 2(5.4 \text{ cm}) = 15 \text{ cm}$

Area: $bh = (5.4 \text{ cm})(2 \text{ cm}) = 10.8 \text{ cm}^2$



Now draw a congruent parallelogram.

Practice Test

Use your ruler (use cm) to determine the dimensions, and then calculate the perimeter and area of each shape below. Round to the nearest tenth of a cm.

Base: 6.7 cm

Height: 5.3 cm

Perimeter: $5.6 + 7.1 + 6.7$

$= 19.4 \text{ cm}$

Area:

$$\hookrightarrow \frac{1}{2}bh = \frac{1}{2}(6.7)(5.3) = \underline{17.8 \text{ cm}^2}$$

Now draw a similar triangle that is only half the size of the original.

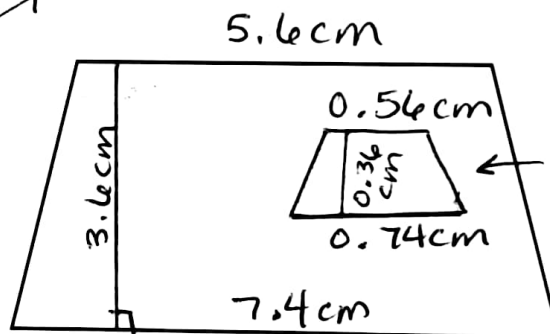
$$A = \frac{1}{2}(5.6 + 7.4)(3.6) = \underline{23.4 \text{ cm}^2}$$

Base 1: 5.6 cm

Height: 3.6 cm

Base 2: 7.4 cm

Area $= \frac{1}{2}(b_1 + b_2)h$



Now draw a similar trapezoid *inside* the original of any scale factor.

$$A_{\text{total}} = bh + \frac{1}{2}\pi r^2$$

$$= (5.4)(3) + \frac{1}{2}(3.14)(2.7)^2$$

$$\text{Area} = 2.4 + 11.4 = \underline{13.8 \text{ cm}^2}$$

Length: 5.4 cm

Width: Rectangle: 3 cm

Radius = 2.7 cm

Perimeter: $5.4 + 2(3) + \pi(2.7)$

$$= \underline{19.9 \text{ cm}}$$

Area:

$$5.4 \boxed{3} + \text{half circle}$$

